The Signalling Channel of Negative Interest Rates[*]

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Abstract

Negative policy rates can convince markets that deposit rates will remain lower-for-longer, even when current deposit rates are constrained by zero. This is the signalling channel of negative interest rates. We analyse the optimality and effectiveness of negative rates in the context of this novel transmission channel. In a stylized model, we prove two necessary conditions for optimality: time-consistency and a preference for policy smoothing. In an estimated model, we show the signalling channel dominates banks' costly interest margin channel. However, the effectiveness of negative rates depends sensitively on the degree of policy inertia, level of reserves, and ZLB duration.

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1 Introduction

In recent years, negative interest rates have become an additional policy tool for some central banks while others have kept policy rates positive, despite a need for further monetary easing. In the euro area, the deposit facility rate—paid on bank reserves at the European Central Bank (ECB)—and the EONIA interbank market rate turned negative in June 2014 (Figure 1(a)).[1] In response, average household deposit rates declined but remained positive with the fraction of deposits paying zero interest rising but virtually no banks passing the negative reserve rate on to household depositors (Figure 1(b)). At the same time, and despite negative interest margins, banks started to accumulate massive excess reserves with total reserves rising to over 20% of deposits by 2018 (Figure 1(c)).[2]

This raises four questions: i) Given that banks do not (or cannot) pass on negative rates to households, what is the transmission channel through which they operate? ii) What are the consequences of a negative rate policy for the banking system and credit creation if a large fraction of banks' assets are reserves that earn a negative return? iii) When both direct and indirect effects are accounted for, are negative rates effective in raising aggregate demand? iv) Under what conditions should negative rates be in an optimal policymaker toolkit? This paper studies the interplay of a contractionary bank interest margin channel and a novel expansionary *signalling* channel to address these questions.

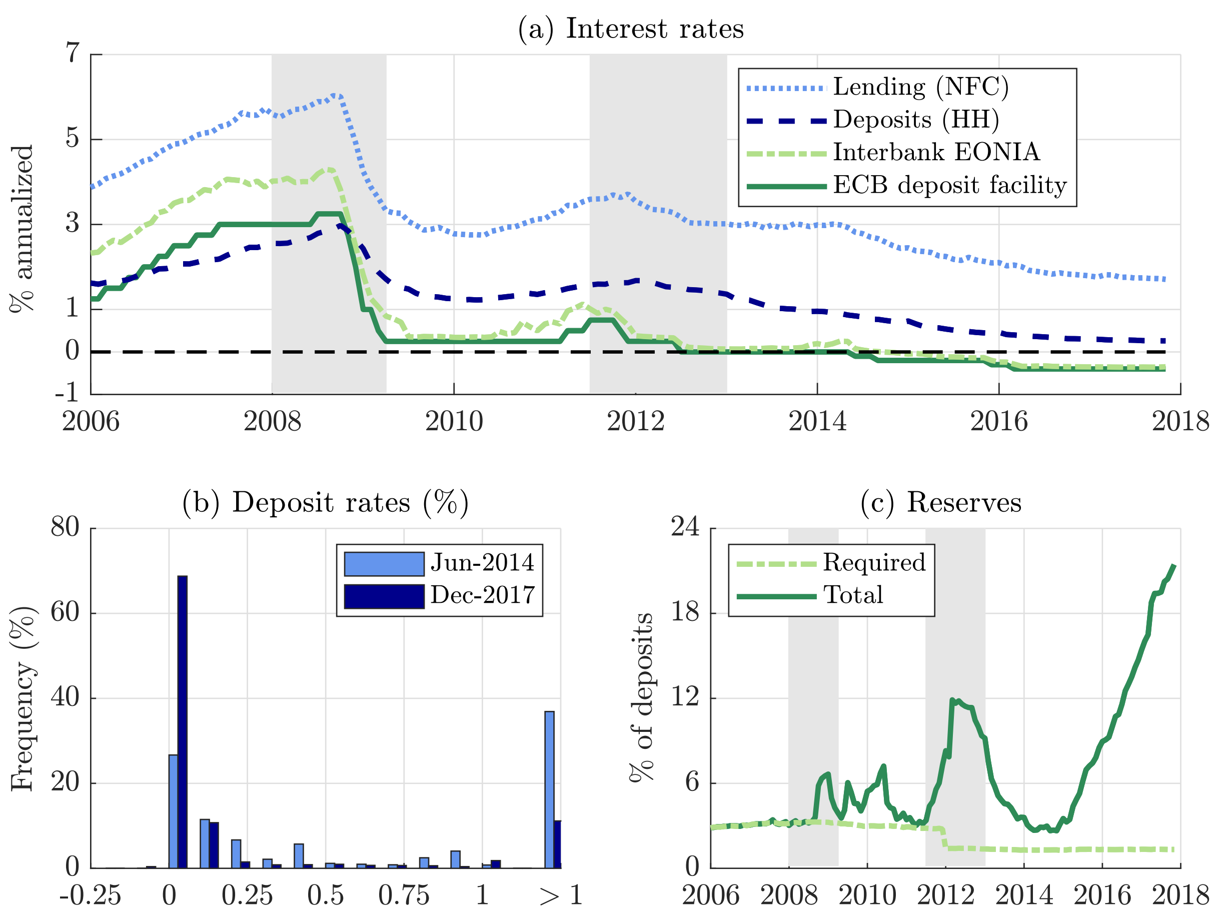
Our first contribution is to analytically explore this signalling channel by which negative reserve rates can be expansionary, even when deposit rates are constrained. We build a stylized model in which banks hold central bank reserves and monetary policy can set a negative reserve rate, but—in line with empirical evidence—household deposit rates are constrained by a zero lower bound (ZLB). Away from the ZLB, the behavior of the model is isomorphic to a model without reserves. However, when the central bank sets a negative reserve rate, the deposit rate, that enters the household Euler equation, remains at zero, creating a wedge between the return on reserves and the cost of bank funding (deposits). All else equal, a negative reserve rate acts like a contractionary bank net worth shock ("the costly interest margin channel of negative interest rates"), where the shock is scaled by the total amount of reserves. Credit spreads widen, raising lending rates vis-a-vis deposit rates, and depress investment demand. This direct, or *intra*temporal, effect of negative rates has been a key criticism of negative rate policies by commercial banks.

[1]The ECB subsequently lowered its deposit facility rate to −0.5% in Sep-2019. The Swiss National Bank reached a low of −1.25 to −0.25% for its 3-month LIBOR target range in Jan-2015. Danmarks Nationalbank set its certificates of deposit rate to a low of −0.75% in Feb-2015. The Bank of Japan reached a low of −0.1% for its short-term policy interest rate in Jan-2016. Finally, the Swedish Riksbank lowered its deposit rate to −1.25% and its repo rate to −0.5% in Feb-2016.

[2]While banks' deposit creation and lending determine the reserve-to-deposit ratio and required-to-excess reserve split, the ECB's liquidity and asset purchase programs caused the growth of total reserves.

However, our stylized model shows there can also be substantial expansionary indirect, or *inter*temporal, effects of negative rates that are less directly ascribable to the policy. In particular, we highlight the role negative rates can play in signalling future policy ("the signalling channel of negative interest rates"). This signalling channel—analogous to Bhattarai et al. (2022)'s signalling theory of quantitative easing (QE)—is active when a time-consistent policymaker with a preference for smoothing interest rates uses a negative rate policy as a credible commitment device to keep deposit rates lower-for-longer.

Figure 1: Interest rates and reserves in the euro area



NOTE: In (a) NFC and HH denote non-financial corporation and household composites, respectively; EONIA is the euro area overnight interbank market rate. In (b) deposit rates are on outstanding amounts as reported by individual banks, plotted as a fraction of total deposits in each bucket. In (c) deposits are HH and NFC deposits; excess reserves are total reserves minus required reserves. Source: ECB.

The mechanism underlying the signalling channel is straightforward: When deposit rates are constrained by zero, they will not rise until the reserve rate turns positive. In an environment where the central bank only adjusts rates gradually—due to a preference for smoothing policy—moving the reserve rate into negative territory increases the distance (and the time taken) for it to turn positive again. Thus, a negative reserve rate signals lower-for-longer deposit rates. This signalling is distinct from forward guidance, which is an "open mouth" commitment about future interest rates. In reality, open mouth policy is not always credible. We show—using gradualism as a commitment device—negative rates can create credible forward guidance. Signalling, in our context, therefore derives from time-consistency and smoothing rather than imperfect information as in Melosi (2017).

Given the trade-off between the costly interest margin channel and the signalling channel we use the stylized model to study optimal policy and analytically prove two conditions for the optimal policymaker to include negative rates in its toolkit: i) it sets time-consistent discretionary policy (i.e., cannot commit to future promises) and ii) it has an intrinsic preference for policy smoothing. Under these conditions, a negative reserve rate can act as a tangible signal of maintaining lower future deposit rates. In contrast, a policymaker that can commit to future promises does not use negative rates as it can generate a credible path of deposit rates without incurring the costs of negative rates via the interest margin channel. Equally, a discretionary policymaker without a smoothing preference has no ability to signal and so negative rates generate only a direct cost to banks.

Our second contribution is to study the trade-off between the signalling and costly interest margin channels quantitatively. We develop a medium-scale version of the stylized model, substitute optimal policy for an inertial Taylor-type rule, and estimate the key structural parameters of the model. When monetary policy is described by an inertial rule, setting a negative policy rate signals lower-for-longer deposit rates, depressing post-ZLB deposit rates and potentially extending the overall ZLB duration. Even with current deposit rates unchanged, this generates an expansionary intertemporal aggregate demand effect.

In our estimated model, we show quantitatively that the contractionary intratemporal effect of negative rates via the costly interest margin channel is more than offset by the expansionary intertemporal aggregate demand effect via the signalling channel. In general equilibrium, asset values increase and banks benefit from capital gains. This reverses the fall in net worth due to the costly interest margin channel, compresses credit spreads (lowering lending rates), and boosts investment demand. We illustrate this with a novel decomposition of bank profits, highlighting the role of capital gains. We also show the effectiveness of negative rates (relative to standard policy) depends on three key factors: i) more policy inertia strengthens the expansionary signalling channel, ii) more reserves in the banking system magnifies the costly interest margin channel, and iii) a longer expected ZLB duration depresses the expansionary signalling channel and magnifies the costly interest margin channel. Finally, we show our results are robust to changes in the size of capital gains and not a product of the new-Keynesian forward guidance puzzle.

Literature A growing empirical literature studies the transmission and effectiveness of negative rates. Eisenschmidt and Smets (2018) document that—consistent with our model—euro area banks did not lower household deposits rates below zero. They find negative rates eased financial conditions and created modest credit growth, despite some adverse effects on bank balance sheets. Recent evidence in Altavilla et al. (2022) suggests negative rates did not inhibit the monetary transmission to firm deposit rates. Regarding bank profitability, Altavilla et al. (2018) estimate the impact on bank balance sheets, identify a costly interest margin channel, and find—in line with our paper—that overall negative rates substantially increased banks' asset and equity values. Heider et al. (2019) and Demiralp et al. (2021) show that banks adjust both lending quantity and risk profile in response to negative rates. Girotti et al. (2021) also find evidence for bank portfolio rebalancing and argue negative rates flatten the middle of the corporate loan yield curve. While in our model financial frictions and negative rates determine the volume and not the type of credit extended by banks, we also find that negative rates mainly impact the middle of the yield curve. Thus, a negative rate policy can be seen as a complement to QE (which affects the long end of the yield curve) even if—as our model suggests—negative interest rates are less effective in the presence of QE (through the rise in reserves).

In the theoretical literature, Ulate (2021a,b) investigates monopoly power in the banking sector and shows negative rates are expansionary as monopolistic profit margins allow banks to partly pass-through negative reserve rates. In our model, the banking sector is competitive which suggests our estimates of the expansionary effects of negative rates via the signalling channel—also implicitly active in Ulate (2021b)—may be conservative. Brunnermeier et al. (2022) build a model with partial deposit rate pass-through and show that there can exist a (time-varying) "reversal rate" below which further cuts in the policy rate are contractionary for lending. While this reversal rate can theoretically be positive, in a quantitative model the authors estimate it to be around -1% in the euro area.[3] Abstracting from signalling and a partial rate pass-through, Eggertsson et al. (2022) find negative rates are, at best, ineffective, and at worst contractionary, depending on the parameterization of bank intermediation costs. In our model, the contractionary interest marginal channel microfounds this notion. Onofri et al. (2021) allow households to use non-deposit savings vehicles and banks to use non-deposit funding sources. This feature is key for negative rates to be expansionary in their model.[4] Sims and Wu (2021a,b) study several unconventional monetary policy measures—including negative rates—in a unified framework. Compared to this literature, our paper is the first to characterize optimal

[3] Darracq Pariès et al. (2020) study macroprudential policy in an environment with a reversal rate.

[4] In Onofri et al. (2021), the policy rule is inertial in the notional rather than the actual policy rate, rendering the signalling channel inactive. We provide further details on this in Section 3.1.

policy in a negative rate environment.[5] Further, we explicitly model and thoroughly investigate the trade-off between a novel signalling and a costly interest margin channel. However, there are other potential channels through which negative rates may operate, which we and the literature have not yet studied in detail. Balloch et al. (2022) provide a valuable overview, including search-for-yield effects (from portfolio choice), exchange rate effects (in an open economy setting), and wealth effects (from long-dated assets).

Our paper also contributes to the literature on how to make forward guidance credible. Woodford (2003) shows that, in the absence of commitment, the delegation of policy to a policymaker with an interest rate smoothing objective can be welfare improving. Nakata and Schmidt (2019) demonstrate delegation is even more valuable with occasional episodes at the ZLB.[6] We take this a step further and show delegating to a policymaker with a smoothing preference introduces a new (welfare improving) policy tool—negative rates. In the context of QE, Bhattarai et al. (2022) find QE is effective as the government commits to honour outstanding debt, enabling the discretionary policymaker to generate a credible signal of low future interest rates. Our two papers are highly complementary. Our instrument is negative rates and the commitment device is policy smoothing whereas their instrument is QE and the commitment device is outstanding debt obligations.

A future question is whether the policy smoothing preference (required for signalling) might arise, not intrinsically from delegation, but extrinsically from a feature of the economy. Stein and Sunderam (2018) show policy gradualism under discretion is optimal if the central bank i) has private information and ii) is averse to bond market volatility. However, they do not microfound the policymaker's aversion to the latter. McKay and Wieland (2021) build a model of lumpy durable consumption demand. In their model, a monetary easing increases durable consumption demand today at the expense of tomorrow, forcing policy to remain accommodative for longer. Although beyond the scope of this paper, this feature may give rise to an extrinsic preference for policy smoothing and hence generate a signalling channel of negative interest rates without delegation.

The paper proceeds with Section 2 presenting a stylized model and optimality conditions for negative rates. Section 3 presents a quantitative model and results on the strength of the signalling channel and the effectiveness of negative rates. Section 4 concludes.

[5] Rognlie (2016) studies optimal policy in a model without a banking sector where negative rates can raise aggregate demand but inefficiently subsidize paper currency.

[6] Bonciani and Oh (2023) argue smoothing and QE resolve several new-Keynesian ZLB puzzles.

2 Stylized model and optimal policy

This section sets up a stylized model to qualitatively illustrate the signalling channel of negative interest rates. We show the model can be reduced to a 3-equation new-Keynesian model with an endogenous demand shifter in the IS equation resulting from negative rates and present analytical and numerical results regarding the optimality of negative rates.

2.1 Set up

The model consists of households, banks, firms, and a central bank. Two types of households—savers and borrowers—transact through banks that are subject to lending frictions. Monopolistic firms produce and set prices subject to nominal rigidities. The central bank sets its policy tool—the interest rate on reserves—optimally.

Households Two types of households—savers and borrowers—are distinguished by their relative patience with discount factors β and β_b, respectively, where $0 < \beta_b < \beta < 1$.

A representative saver household is composed of a fraction f workers and $1 - f$ bankers with perfect consumption insurance. Workers and bankers switch with probability $1 - \theta$. When they do, bankers transfer retained profits to the household. Households consume, $C_{s,t}$, supply labor, $L_{s,t}$, and save in bank deposits, D_t, with gross nominal return $R_{d,t}$. Financial markets are incomplete. The saver household's problem is given by

$$V_{s,t} = \max_{\{C_{s,t},L_{s,t},D_t\}} \left(\frac{{C_{s,t}}^{1-\sigma}}{1-\sigma} - \chi \frac{{L_{s,t}}^{1+\varphi}}{1+\varphi} \right) + \beta \exp(s_t) \mathbb{E}_t V_{s,t+1}, \tag{1}$$

subject to

$$P_t C_{s,t} + D_t = P_t W_{s,t} L_{s,t} + R_{d,t-1} D_{t-1} + \Omega_{1,t} - \Omega_{2,t}, \tag{2}$$

where s_t is an AR(1) time-preference shock that generates exogenous movements in the natural real rate, P_t is the aggregate price level, $W_{s,t}$ is the real wage, $\Omega_{1,t}$ are firm and bank profits, and $\Omega_{2,t} = R_{b,t-1} P_{t-1} B_{t-1} - P_t W_{b,t} L_{b,t}$ is a lump-sum transfer from savers to borrowers both households take as given. While contrived, this transfer facilitates a clean set of equilibrium conditions that maintain focus on the key features of the model related to negative interest rates. The first-order conditions are given by $1 = \mathbb{E}_t \Lambda_{t,t+1} R_{d,t} / \Pi_{t+1}$, and $\chi L_{s,t}^{\varphi} = C_{s,t}^{-\sigma} W_{s,t}$, where $\Lambda_{t-1,t} \equiv \beta \exp(s_t) \left(C_{s,t} / C_{s,t-1} \right)^{-\sigma}$ is the saver household's real stochastic discount factor and $\Pi_t \equiv P_t / P_{t-1}$ is the gross inflation rate. A ZLB on the deposit rate, $R_{d,t} \geq 1$, arises as cash offers a zero nominal net return.

The representative borrower household only consists of workers. Its problem is given by

$$V_{b,t} = \max_{\{C_{b,t}, L_{b,t}, B_t\}} \left(\frac{{C_{b,t}}^{1-\sigma}}{1-\sigma} - \chi \frac{{L_{b,t}}^{1+\varphi}}{1+\varphi} \right) + \beta_b \exp(s_t) \mathbb{E}_t V_{b,t+1}, \tag{3}$$

subject to

$$P_t C_{b,t} + R_{b,t-1} P_{t-1} B_{t-1} = P_t W_{b,t} L_{b,t} + P_t B_t + \Omega_{2,t}, \tag{4}$$

where variables have subscript b. B_t are bank loans with gross nominal interest rate $R_{b,t}$. The first-order conditions are given by $C_{b,t}^{-\sigma} = \beta_b e^{s_t} \mathbb{E}_t C_{b,t+1}^{-\sigma} \frac{R_{b,t}}{\Pi_{t+1}}$ and $\chi L_{b,t}^{\varphi} = C_{b,t}^{-\sigma} W_{b,t}$.

Banks The balance sheet of banker j is given by

$$B_t(j) + A_t(j) = D_t(j) + N_t(j), \tag{5}$$

where $N_t(j)$ is net worth and $A_t(j)$ are central bank reserves that earn the gross nominal return R_t. We assume a banker's demand for central bank reserves is given by

$$A_t(j) = \alpha(x_t) D_t(j), \tag{6}$$

where $x_t \equiv R_t / R_{d,t}$, $\alpha(1) = \alpha$, $\alpha(x_t) > 0$, $\alpha'(x_t) > 0$, and $\alpha''(x_t) > 0$. This demand schedule captures the trade-off between banks' preference for holding reserves to self-insure against idiosyncratic liquidity risk and the cost of holding reserves.[7]

Within a period, the timing is as follows: i) Bankers receive loan payments and repay depositors. ii) Bankers exit with probability $1-\theta$. An exiting banker is replaced by a worker with an endowment of net worth, $\bar{N}$. iii) Bankers accept new deposits and demand reserves. iv) A banker can divert a fraction λ of its assets (net of reserves) to its household, in which case, the depositors force bankruptcy and recover the remaining assets.

This agency problem creates a financial friction and makes bankers' net worth a relevant determinant of equilibrium outcomes. The banker problem is given by

$$V_{n,t}(j) = \max_{\{B_t(j), A_t(j), D_t(j), N_t(j)\}} \mathbb{E}_t \Lambda_{t,t+1} \left((1-\theta) N_{t+1}(j) + \theta V_{n,t+1}(j) \right), \tag{7}$$

subject to the banker's balance sheet, (5), reserve demand, (6), incentive compatibility constraint, (8), and net worth accumulation equation, (9), with the latter two given by

$$V_{n,t}(j) \geq \lambda B_t(j), \tag{8}$$

$$N_t(j) = \frac{R_{b,t-1}}{\Pi_t}(1-\tau) B_{t-1}(j) + \frac{R_{t-1}}{\Pi_t} A_{t-1}(j) - \frac{R_{d,t-1}}{\Pi_t} D_{t-1}(j), \tag{9}$$

[7] A simple model to microfound this functional form is presented in Appendix A.1. For other models of idiosyncratic liquidity risk and reserve demand see Güntner (2015) and Bianchi and Bigio (2022).

where the tax $\tau \equiv 1 - \frac{\beta_b}{\beta}$ ensures the steady state is undistorted by the financial friction. The central bank sets the reserve rate and supplies reserves elastically. Since banks are competitive, arbitrage ensures $R_t = R_{d,t}$ when $R_{d,t} > 1$. In a symmetric equilibrium, bankers have a common leverage ratio, denoted $\Phi_t \equiv B_t/N_t = B_t(j)/N_t(j)$. The banking sector can thus be summarized in two equations.[8] Aggregate net worth is given by

$$N_{t+1} = \theta \left(\frac{R_{b,t}}{\Pi_{t+1}} (1-\tau) \Phi_t - \frac{R_{d,t} - \alpha(x_t) R_t}{(1-\alpha(x_t)) \Pi_{t+1}} (\Phi_t - 1) \right) N_t + (1-\theta) \bar{N}, \tag{10}$$

and, if the incentive constraint binds, (11) holds, otherwise arbitrage ensures (12) holds,

$$\lambda \Phi_t = \mathbb{E}_t \Lambda_{t,t+1} \frac{1 - \theta + \theta \lambda \Phi_{t+1}}{\Pi_{t+1}} \left(R_{b,t} (1-\tau) \Phi_t - \frac{R_{d,t} - \alpha(x_t) R_t}{1 - \alpha(x_t)} (\Phi_t - 1) \right), \tag{11}$$

$$R_{b,t} (1-\tau) = \frac{R_{d,t} - \alpha(x_t) R_t}{1 - \alpha(x_t)}. \tag{12}$$

Production Intermediate firm i produces output $X_t(i) = L_{s,t}(i)^{\omega} L_{b,t}(i)^{1-\omega}$, hiring workers in a competitive labor market. Retail firms repackage intermediate output one-for-one, $Y_t(i) = X_t(i)$. Final output, $Y_t = \left(\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)}$, is a CES aggregate of retail firm output, where $\epsilon > 0$. Cost minimization results in demand for good i given by $Y_t(i) = (P_t(i)/P_t)^{-\epsilon} Y_t$, where $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{1/(1-\epsilon)}$. Subject to a Calvo nominal price rigidity, each period, retail firms adjust their prices with probability $1 - \iota$. In doing so, they solve $\max_{P_t(i)} \mathbb{E}_t \sum_{\tau=0}^{\infty} \iota^{\tau} \Lambda_{t,t+\tau} \left(\frac{P_t(i)}{P_{t+\tau}} - \mathcal{M}_{t+\tau} \right) Y_{t+\tau}(i)$ subject to the demand for good i, where $\mathcal{M}_t = W_{s,t}^{\omega} W_{b,t}^{1-\omega} / \left(\omega^{\omega} (1-\omega)^{1-\omega} \right)$ denotes marginal cost.

The aggregate resource constraint is given by $C_{s,t} + C_{b,t} = Y_t$. To close the model, the central bank sets the reserve rate, R_t, optimally as described in Section 2.3.

2.2 Log-linear equilibrium

The beauty of this stylized model is that its log-linear form is very similar to the canonical 3-equation new-Keynesian model.[9] In the following, we focus on the case when $\theta = 0$ where bankers survive a single period. In this case, when the financial sector incentive compatibility constraint binds, the private-sector equilibrium conditions are given by

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t, \tag{13}$$

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1 - \mathsf{c}}{\sigma} (r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t) - \mathsf{c} (\mathbb{E}_t \phi_{t+1} - \phi_t), \tag{14}$$

$$\phi_t = \tau_1 \mathbb{E}_t \phi_{t+1} - \tau_2 (r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t) - \tau_3 (r_{d,t} - r_t), \tag{15}$$

[8] A full derivation of the banker's problem for the quantitative model can be found in Appendix B.1.

[9] Appendix A.2 provides the full derivation of the log-linear model.

where lower-case letters are the log-levels of their upper-case counterparts and c is the steady state consumption share of borrowers. The other parameters are given by

$$\kappa = \frac{(1-\iota\beta)(1-\iota)(\varphi+\sigma)}{\iota}, \ \tau_1 = \frac{\Phi\sigma}{1+\Phi\sigma}, \ \tau_2 = \frac{\Phi}{1+\Phi\sigma}, \ \tau_3 = \frac{\Phi-1}{1+\Phi\sigma}\frac{\alpha}{1-\alpha}.$$

Equation (13) is the standard Phillips curve. Since we only have time-preference shocks, output and output gap coincide. Equation (14) is the IS curve. When $\mathsf{c} = 0$, it reduces to the standard IS curve. Two more points are worth making: First, the deposit rate, $r_{d,t}$, rather than the policy rate, r_t, enters the IS curve. Second, the IS curve features an endogenous demand shifter, $\mathsf{c}\left(\mathbb{E}_t\phi_{t+1} - \phi_t\right)$, which result from leverage fluctuations. Equation (15) is the banks' incentive constraint and determines leverage. Its final term captures the costly interest margin channel: When the reserve and deposit rates deviate, inefficient fluctuations in bank leverage feed through into aggregate demand fluctuations.

The costly interest margin channel even operates in the absence of financial frictions because banks, active in a competitive environment, still need to break-even.[10] When the incentive constraint does not bind, (15) disappears and (14) can be rewritten as

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(r_{d,t} - \mathbb{E}_t\pi_{t+1} - s_t\right) - \phi\left(r_{d,t} - r_t\right), \tag{16}$$

where $\phi \equiv (\mathsf{c}/\sigma)\,\alpha/\left(1-\alpha\right)$. In this case, endogenous demand shifts in the IS curve occur when the reserve rate deviates from the deposit rate. All else equal, when the reserve rate, r_t, turns negative and the deposit rate, $r_{d,t}$, is bounded by zero, this pushes down output, y_t. To see why, we can write the credit spread, $r_{b,t} - r_{d,t}$, as

$$r_{b,t} - r_{d,t} = \frac{\alpha}{1-\alpha}\left(r_{d,t} - r_t\right). \tag{17}$$

When $r_t < r_{d,t}$, banks pass on the cost of the negative rate policy into higher borrowing rates, $r_{b,t}$, resulting in a reduction in consumption demand by borrowers. The pass-through from negative rates to the credit spread—the *costly interest margin channel*—is increasing in the quantity of reserves in the banking system, α.

PROPOSITION 1 *The costly interest margin channel of negative interest rates is exacerbated by an increase in the quantity of reserves, α, in the banking system.*

This result is analogous to tax theory, with the reserve rate as a tax and the quantity of reserves as the tax base. It illustrates why a negative rate policy may be contractionary.

[10] In the quantitative model in Section 3, we ensure the incentive constrain binds and the financial accelerator operates. In this section we pursue the frictionless case as it provides clean analytical insights.

2.3 Analytical results

This section studies the conditions for negative interest rates to be a tool in an *optimal* policymaker's toolkit and introduces our theory of the signalling channel.[11]

Optimal policy To study optimal policy, we assume social welfare is approximated by a quadratic function in inflation and the output gap,

$$V_t^{SW} = -\frac{1}{2}\left(\pi_t^2 + \lambda y_t^2\right) + \beta \mathbb{E}_t V_{t+1}^{SW}, \tag{18}$$

where $\lambda = \frac{\kappa}{\epsilon}$. A model-consistent welfare function depends on the welfare weights of savers and borrowers. For tractability, we use a policy-relevant function consistent with i) the microfounded welfare function of the canonical new-Keynesian model, and ii) many central banks' dual mandate. The policymaker maximizes (18)—setting the reserve rate, r_t—subject to the private-sector equilibrium conditions, and three constraints given by

$$r_{d,t} \geq 0, \qquad r_{d,t} - r_t \geq 0, \qquad r_{d,t}\left(r_{d,t} - r_t\right) = 0. \tag{19}$$

The first constraint in Equation (19) is the ZLB on the deposit rate. The second states the deposit rate cannot be below the reserve rate. The third ensures the reserve and deposit rate can only diverge when the deposit rate is at zero. In sum, while the reserve rate can turn negative, away from the ZLB on the deposit rate, arbitrage equates the two.

In the following, we consider an optimal policymaker that maximizes Equation (18), first, under full commitment, and, second, in a time-consistent (discretionary) manner.

Proposition 2 *Under commitment—when the policymaker solves for a state-contingent plan* $\{\pi_t, y_t, r_t, r_{d,t}\}_{t=0}^{\infty}$ *by maximizing* (18) *subject to the sequence of constraints* (13), (16), (19)*—it follows that* $r_t \geq 0 \; \forall \; s_t$.

Proof See Appendix A.4. ■

Proposition 2 states that with full commitment, a policymaker will never use negative interest rates. The intuition is relatively simple. Under commitment, the central bank can credibly promise to hold the deposit rate lower-for-longer in the future in order to compensate, in part, for the presence of the ZLB. Setting a negative reserve rate results in a cost via the interest margin channel without any further benefit.

[11] Appendix A.3 reports the behaviour of the model under a simple Taylor-type rule.

Proposition 3 *Under discretion—when the policymaker solves for* $\{\pi_t, y_t, r_t, r_{d,t}\}$ *re-optimizing (18) every period subject to (13), (16), (19), and the actions of future policymakers—it follows that* $r_t \geq 0 \; \forall \; s_t$.

Proof See Appendix A.4. ■

Proposition 3 states that negative rates are also not part of the optimal time-consistent policymaker's toolkit. Under discretion, the policymaker cannot commit to future actions and so a negative rate does not signal lower rates in the future. Setting a negative reserve rate results in a cost via the interest margin channel without any further benefit.

Propositions 2 and 3 suggest that negative rates are never optimal in an environment with deposit rates bounded by zero and no alternative transmission to lending rates—for example, through monopolistic competition among banks as in Ulate (2021b). However, welfare can sometimes be raised by appointing a central banker whose preferences do not coincide with the social welfare function (Rogoff, 1985). In the following, we show that delegating policy to a central banker that places a weight on smoothing policy will—under certain conditions—allow negative rates to increase welfare. With a preference for smoothing interest rates, lowering policy rates today signals lower policy rates tomorrow. This is the essence of the expansionary *signalling channel of negative interest rates.*

Technically, smoothing gives the policymaker an endogenous state variable that allows it to signal. Whether another state variable in the model structure will do the job is an open question. However, not any endogenous state variable will do. Propositions 2 and 3 also hold when a hybrid Phillips curve makes lagged inflation a state.

Optimal policy with delegation Woodford (2003) and Nakata and Schmidt (2019), amongst others, show that under discretion delegating policy to a policymaker with a preference for smoothing can be desirable. We therefore introduce a delegated central bank loss function (that deviates from the social welfare function) given by

$$V_t = -\frac{1}{2}\left((1-\psi)\left(\pi_t^2 + \lambda y_t^2\right) + \psi\left(r_t - r_{t-1}\right)^2\right) + \beta \mathbb{E}_t V_{t+1}, \tag{20}$$

with a preference for interest rate smoothing weighted by $\psi \in (0, 1)$. Proposition 4 states a set of necessary conditions for negative rates to be in the optimal policymaker's toolkit.

Proposition 4 *Two necessary conditions for the optimality of negative interest rates in this framework are i) a discretionary policy setting, and ii) the delegation of policy to a policymaker with a preference for smoothing interest rates* ($\psi > 0$).

The first necessary condition prevents the policymaker from exploiting "open-mouth" forward guidance to ease policy at the ZLB. The second enables the policymaker to use a change in the current level of the policy rate, r_t, to signal a change in future deposit rates.

The intuition for Proposition 4 is as follows. The discretionary policymaker reoptimizes every period, taking the policy functions of future policymakers as given. When $\psi > 0$, r_{t-1} becomes an endogenous state variable making negative rates a tangible signal of future rates in a time-consistent equilibrium. To be more precise—conditional on the "regime" the reserve rate r_t is in—when maximizing (20) subject to (13), (14), and (19) the first-order conditions of the optimal policy problem can be written as follows:

$$
\begin{aligned}
&\texttt{Regime I:}\ (r_t > 0) \\
&0 = \psi (1+\beta) r_t - \psi r_{t-1} - \psi\beta\mathbb{E}_t r_{t+1} + (1-\psi)\beta\mathbb{E}_t \frac{\partial \pi (r_t, s_{t+1})}{\partial r_t} \\
&\quad + (1-\psi)\left(\mathbb{E}_t \frac{\partial y (r_t, s_{t+1})}{\partial r_t} + \sigma^{-1}\mathbb{E}_t \frac{\partial \pi (r_t, s_{t+1})}{\partial r_t} - \sigma^{-1}\right)(\lambda y_t + \kappa \pi_t), \qquad r_{d,t} = r_t. \\
&\texttt{Regime II:}\ (r_t < 0) \\
&0 = \psi (1+\beta) r_t - \psi r_{t-1} - \psi\beta\mathbb{E}_t r_{t+1} + (1-\psi)\beta\mathbb{E}_t \frac{\partial \pi (r_t, s_{t+1})}{\partial r_t} \\
&\quad + (1-\psi)\left(\mathbb{E}_t \frac{\partial y (r_t, s_{t+1})}{\partial r_t} + \sigma^{-1}\mathbb{E}_t \frac{\partial \pi (r_t, s_{t+1})}{\partial r_t} + \phi\right)(\lambda y_t + \kappa \pi_t), \qquad r_{d,t} = 0. \\
&\texttt{Regime III:}\ (r_t = r_{d,t} = 0),
\end{aligned}
$$

where $\pi_t = \pi(r_{t-1}, s_t)$, for example, denotes the solution for inflation as a function of the state vector. In any period, the economy can be in three possible regimes. `I`: The ZLB does not bind, `II`: The deposit rate ZLB binds and the reserve rate is set negative, or `III`: The deposit rate ZLB binds and the reserve rate is set to zero. `Regime III` allows for the possibility that a negative rate policy is feasible but not optimal. For example, we will see that if ψ is sufficiently small or ϕ is sufficiently large, `Regime II` is never visited and at the ZLB the reserve rate is kept at zero. The first-order condition also illustrates the role of policy smoothing in generating the signalling channel. When $\psi = 0$, it reduces to a static condition: $y_t = -\frac{\kappa}{\lambda}\pi_t$. When $\psi > 0$, the policymaker takes account of the actions of future policymakers and past actions influence current decisions.

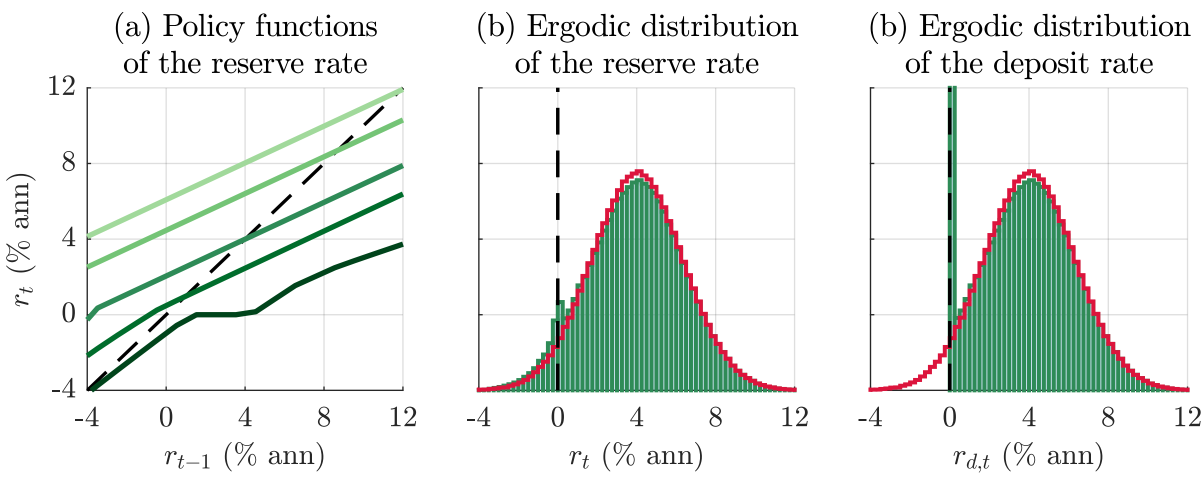
2.4 Numerical exercise and comparative statics

The previous section showed negative rates can be optimal when policy is discretionary and delegated to a policymaker with a preference for smoothing policy. This section illustrates their optimal use with a numerical exercise and comparative statics.

Numerical exercise We solve the model with the Endogenous Grid Method and take parameter values from Nakata and Schmidt (2019).[12] Risk aversion is $\sigma = 0.5$, the discount factor of savers is $\beta = 0.99$, the Phillips curve slope is $\kappa = 0.008$, and the output gap weight in welfare is 7.85×10^{-4}. In addition, we set the consumption share of borrowers to $\mathsf{c} = 0.4$ and the natural rate, s_t, follows an AR(1) process with persistence 0.85 and variance 0.0016, approximated using the Tauchen quadrature algorithm with 21 nodes.

We set the reserve-to-deposit ratio to $\alpha = 0.2$, implying $\phi = 0.2$. All else equal, a 25 basis point (bp) gap between the deposit and reserve rate widens the output gap by 5bps. The smoothing preference, $\psi = 0.029$, is set to maximize welfare in the absence of negative rates.[13] Since ϕ and ψ are crucial for the strength of the signalling and costly interest margin channel, respectively, we illustrate the comparative statics of changing both below.

Figure 2: Optimal policy solution



NOTE: (a) plots policy functions for five different s_t values. The black-dash is the 45-degree line. (b) and (c) plot ergodic distributions generated from simulations of length 10^6 with a burn-in of 10^3. The filled-green plots the distribution with negative rates, the red line the distribution without a ZLB.

Figure 2(a) plots policy functions $r_t = r(r_{t-1}, s_t)$, and reveals three new insights into optimal discretionary policy with smoothing. First, the policy functions turn negative, illustrating that an optimal policymaker uses negative rates in these conditions. Second, there are "inaction" regions where the policy function is flat. For a large fall in s_t, the policymaker initially drops the reserve rate to zero and only subsequently sets it negative.

[12] Appendix A.6 describes the algorithm to solve the time-consistent optimal policymaker's problem.

[13] Appendix A.7 derives the consumption equivalent welfare measure and shows it is hump-shaped and concave in the smoothing parameter, ψ. Thus, policy is optimally delegated to a policymaker with a positive but finite smoothing preference. Appendix A.8 shows the optimal response to a natural rate shock.

Third, the policy function slope is steeper to the left of the inaction region, suggesting that once the policymaker passes into negative territory, it will cut the reserve rate more aggressively than if unconstrained by the ZLB. Proposition 6 (below) rationalizes this.

Panels (b) and (c) display the ergodic distributions (in green) for r_t and $r_{d,t}$, respectively. The deposit rate distribution is truncated by the ZLB whereas the reserve rate has some mass below zero. Due to the observed inaction, the r_t distribution is non-symmetric, with more mass at both $r_t = 0$ and $r_t < 0$ relative to the unconstrained distribution (red line). Comparing the distributions with and without negative rate policies, the deposit rate is expected to bind 4.4% and 3.7%, respectively. This increased frequency at the ZLB with negative rates is welfare improving. With negative rates, households would forgo 2.33% of consumption per period to avoid uncertainty, compared to 2.57% without negative rates.

Comparative statics We make two further assumptions: i) s_t becomes iid; ii) the policymaker disregards the output gap ($\lambda = 0$) and only cares about smoothing policy between periods 2 and 1. This effectively reduces the model to a 2-period problem since $\{\pi_t, y_t\} = \{0, 0\}$ for $t \geq 3$, allowing for closed-form solutions. First, we derive two additional analytical results to complement Proposition 4 in this environment.

Proposition 5 *There exists a threshold, ϕ^*, for the cost of negative rates, ϕ, such that a negative interest rate policy raises both inflation and output if and only if* $\phi < \phi^* \equiv \frac{\psi}{\psi + (1-\psi)(\kappa\sigma^{-1})^2} \times \sigma^{-1}\left(1 + \kappa\sigma^{-1}\right)$.

Proof See Appendix A.9. ■

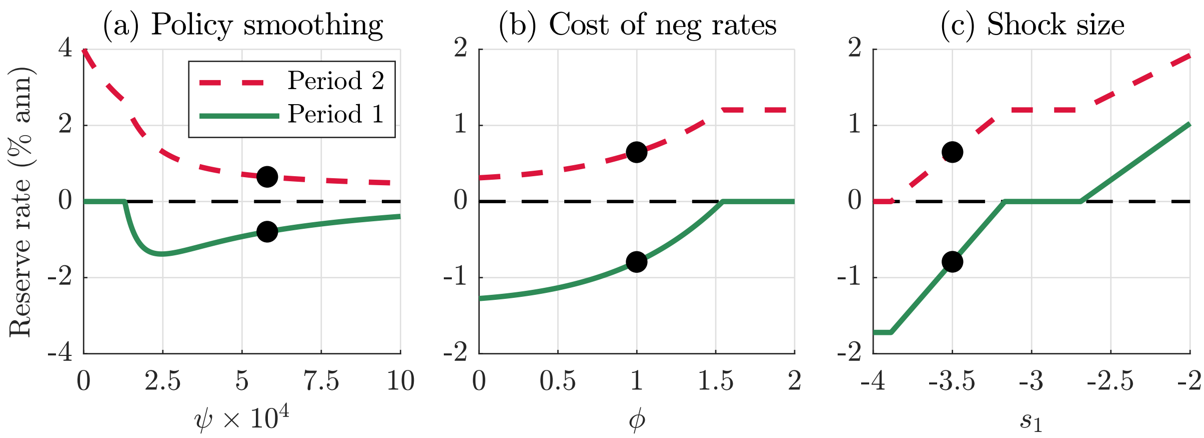
Proposition 5 shows that the threshold on the cost of negative rates is monotonically increasing in the degree of policy smoothing, ψ. For any $\phi < \phi^*$, the marginal cost of lowering the reserve rate in negative territory, ϕ, is lower than the marginal benefit in terms of inflation and output, given by the product of two terms: i) the optimal marginal effect of lowering today's policy rate on tomorrow's rate (when $\psi = 0$, signalling is inactive and $\phi^* = 0$); and ii) today's effect of tomorrow's marginally lower rate via expected inflation and output. Thus, intuitively, Proposition 5 can also be interpreted as a sufficient condition for negative rates to be welfare improving.

Proposition 6 *For $\phi < \phi^*$, the response of policy in negative territory to offset a marginal change in the natural rate ($dy_1/ds_1 = 0$) is given by* $\left.\frac{\partial r_1}{\partial s_1}\right|_{r_1<0} > \left.\frac{\partial r_1}{\partial s_1}\right|_{r_1>0}$.

Proof See Appendix A.9. ■

Proposition 6 states that, conditional on rate cuts being effective ($\phi < \phi^*$), the cut in the policy rate needed to generate the same effect on output is larger in negative than in positive territory.[14] Intuitively, away from the ZLB, a marginal cut in the policy rate has an additional direct benefit on output of σ^{-1} via the IS equation. In contrast, in negative territory, the overall effect of a cut is smaller because the only direct effect is the cost $-\phi$.

Figure 3: Optimal policy sensitivity analysis



NOTE: The black-dot refers to the baseline parameterization across the three panels with the weight on policy smoothing scaled down by 50 ($\psi = \psi/50$) and the cost of negative rates scaled up by 5 ($\phi = \phi \times 5$). The period 1 natural rate, s_1, is set to -3.5. We rescale these parameters to visually highlight the trade-offs at play. This adjustment is not necessary for the qualitative nature of the results but allows to zoom in on a range of the parameter space that captures the full spectrum of comparative statics results.

We conclude with three comparative statics exercises. Figure 3(a) illustrates the non-monotonic effect of varying the smoothing parameter, ψ, on the optimal period-1 reserve rate, r_1, when the natural rate is deeply negative. Consistent with Proposition 5, when ψ is low, the benefit of signalling is outweighed by the cost of negative rates and the reserve rate is set to zero. As the smoothing parameter increases, however, negative rates become optimal. With $r_{d,1}$ at zero, lowering r_1 lowers r_2, raises expected inflation and lowers the real interest rate. With a moderate ψ, the policymaker needs a very negative rate to lower r_2, but, as ψ increases, signalling becomes more powerful and a smaller decline is sufficient to achieve the same fall in r_2, resulting in the observed non-monotonicity.

Panel (b) shows r_1 is increasing and convex in the cost parameter, ϕ. From the Phillips curve, inflation is linear in ϕ, which means welfare is quadratic in ϕ. Thus, as ϕ increases, the policymaker rapidly reduces the degree of negative rates it willingly deploys. Finally,

[14]The inequality in the proposition holds equally for inflation with $d\pi_1/ds_1 = 0$.

Panel (c) varies the natural rate, s_1. As s_1 falls, the policy rate falls to accommodate it. However, there exists an inaction region where the policy rate is zero. Only for sufficiently large shocks does the policymaker use negative rates. Consistent with Proposition 6, the slope $(\partial r_1/\partial s_1)$ is steeper to the left of the inaction region.

3 Quantitative model

The previous section set up a stylized model to qualitatively study the optimality of negative rates. This section develops a richly specified and carefully estimated medium-scale model to quantitatively assess their effectiveness and the relative strength of the costly interest margin and signalling channels. Using a novel decomposition of bank profits and various model modifications, we further elucidate the transmission of negative rates.

3.1 Set up

The basis of the model is a financial-friction new-Keynesian model as in Gertler and Karadi (2011). In contrast to our stylized model in Section 2, we dispense with borrower households and instead have firms borrowing from banks to finance the rent of capital. We introduce endogenous capital formation, investment adjustment costs, consumption habits, and, instead of studying optimal policy, we endow the central bank with an inertial Taylor-type rule to set the reserve rate. For compactness, rather than specifying the entire model below, we only focus on features that differ markedly from the stylized model.[15]

Households Three changes are made to households. One, only a representative (saver) household exists. Two, preferences exhibit consumption habits, $\tilde{C}_t \equiv C_t - \hbar C_{t-1}$. Three, we introduce a Smets and Wouters (2007) AR(1) risk premium shock, ζ_t, to generate the ZLB scenario.[16] Thus, the household problem is given by

$$V_t = \max_{\{C_t, L_t, D_t\}} \left(\log \tilde{C}_t - \frac{\chi}{1+\varphi} {L_t}^{1+\varphi} \right) + \beta \mathbb{E}_t V_{t+1}, \tag{21}$$

subject to

$$P_t C_t + D_t = P_t W_t L_t + \exp\left(\zeta_{t-1}\right) R_{d,t-1} D_{t-1} + \Omega_t. \tag{22}$$

[15] Appendix B.1 derives the financial sector equations and B.2 lists the full set of equilibrium conditions.

[16] While both risk premium and discount factor shocks are common in the literature to induce a demand-driven ZLB scenario, the risk premium shock is preferable in a model with endogenous capital formation as it induces a positive co-movement of consumption and investment.

Bankers We make three changes to the banking sector. One, we assume banker j buys $S_t(j)$ units of firm equity at price Q_t (rather than lending to borrower households). Firm equity pays a stochastic real return, $R_{k,t+1}$. Thus, the banker solves

$$V_{n,t}(j) = \max_{\{S_t(j),A_t(j),D_t(j),N_t(j)\}} \mathbb{E}_t\Lambda_{t,t+1}\left((1-\theta)N_{t+1}(j) + \theta V_{n,t+1}(j)\right), \tag{23}$$

subject to

$$Q_tS_t(j) + A_t(j) = D_t(j) + N_t(j), \tag{24}$$

$$V_{n,t}(j) \geq \lambda Q_tS_t(j), \tag{25}$$

$$A_t(j) = \alpha(x_t)D_t(j), \tag{26}$$

$$N_t(j) = R_{k,t}Q_{t-1}S_{t-1}(j) + (R_{t-1}/\Pi_t)A_{t-1}(j) - (R_{d,t-1}/\Pi_t)D_{t-1}(j). \tag{27}$$

The incentive constraint always binds in our baseline parameterization. Two, in equilibrium, $S_t = K_t$, where $S_t = \int_j S_t(j)dj$ and K_t is aggregate capital. Three, exiting bankers are replaced by workers with initial net worth equal to a fraction ω of total firm equity in the previous period. Hence, the evolution of aggregate net worth is given by

$$N_t = \theta\left(R_{k,t}\Phi_{t-1} - \frac{R_{d,t-1} - \alpha(x_t)R_{t-1}}{(1-\alpha(x_t))\Pi_t}(\Phi_{t-1}-1)\right)N_{t-1} + \omega Q_tK_{t-1}. \tag{28}$$

Capital goods firms Capital goods firms are new to the model, repair depreciated capital, and produce new capital. Existing capital depreciates at rate δ and is refurbished at unit cost. New capital, $K_{n,t}$, is produced using technology $K_{n,t} = f(I_{n,t}, I_{n,t-1})$, where $I_{n,t}$ is investment in new capital formation. Accordingly, capital goods firms solve $V_{k,t} = \max_{I_{n,t}}(Q_tK_{n,t} - I_{n,t}) + \mathbb{E}_t\Lambda_{t,t+1}V_{k,t+1}$. Production technology comes with quadratic adjustment costs, $f(\cdot) \equiv \left(1 - (\eta/2)\left((I_{n,t}+I)/(I_{n,t-1}+I) - 1\right)^2\right)I_{n,t}$, where $I = \delta K$ is defined as steady state gross investment given by $I_t = f(I_{n,t}, I_{n,t-1}) + \delta K_{t-1}$. Thus, capital accumulation follows $K_t = K_{t-1} + f(I_{n,t}, I_{n,t-1})$.

Intermediate goods firms Intermediate goods firms produce and sell intermediate output, $Y_t = K_{t-1}^{\gamma}L_t^{1-\gamma}$, at price $P_{m,t}$. Their labor demand is $W_t = P_{m,t}(1-\gamma)Y_t/L_t$ and profits per unit of capital are $P_{m,t}\gamma Y_t/K_{t-1}$. Capital is purchased using external finance. At the start of the period, firms issue S_t units of equity to bankers at price Q_t. In return, bankers receive next period's realized return per unit of capital, $R_{k,t} = \frac{P_{m,t}\gamma Y_t/K_{t-1} + Q_t - \delta}{Q_{t-1}}$.

Other Retail firms are unchanged except we introduce a cost-push shock by making the elasticity of substitution between goods time-varying. The aggregate resource constraint is $Y_t = C_t + I_t + G$, where G is exogenous government spending, set at $G/Y = 0.2$.

Monetary policy The central bank sets the reserve rate, which when unconstrained follows a Taylor-type inertial policy rule given by

$$R_{T,t} = \left(R\,\Pi_t^{\phi_\pi}\left(X_t/X\right)^{\phi_x}\right)^{1-\rho} R_{t-1}^{\rho}\exp\left(\varepsilon_{m,t}\right), \tag{29}$$

where $R_{T,t}$ is the rate implied by the policy rule, $X_t = 1/\mathcal{M}_t$ is the mark-up and—in the absence of sticky wages—a good proxy of the output gap, and $\varepsilon_{m,t}$ is an iid shock. The degree of inertia is given by ρ and the inertial term is the lagged reserve rate. The policy rule is not inertial when the policy rate is bounded at zero.[17] In what follows, we compare three policy scenarios equivalent to the regimes in Section 2 and Ulate (2021b):

I. The unconstrained ("No ZLB") scenario, in which both the reserve and deposit rate are unconstrained and can turn negative, is given by $R_t = R_{d,t} = R_{T,t}$.

II. The deposit rate-only ZLB ("ZLB: R_d only") scenario—our baseline to study the effects of negative interest rates—in which the deposit rate is bounded by zero, but the reserve rate can turn negative, is given by $R_t = R_{T,t}$ and $R_{d,t} = \max\{1, R_{T,t}\}$.

III. The standard ZLB scenario ("ZLB: R_d & R"), in which both the reserve and deposit rate are constrained by zero, is given by $R_t = R_{d,t} = \max\{1, R_{T,t}\}$.

3.2 Parameterization

Table 1 summarizes our baseline parameter values, distinguishing between those that are standard (Block A), calibrated (B), and estimated (C). The model is set up at quarterly frequency. We solve the model using the Guerrieri and Iacoviello (2015) toolkit—a piece-wise first-order perturbation approach—to account for the occasionally binding ZLB. The sensitivity and robustness of our main results is extensively documented in Section 3.4.

Block A parameters are assigned standard values from the literature. Consumption habits are $\hbar = 0.815$ and the inverse Frisch elasticity is set to $\varphi = 0.276$, a relatively low value we pick as a stand-in for nominal wage rigidities in the model. The elasticity of substitution is $\epsilon = 4.167$, in line with a steady state mark up of around 30%. The Calvo parameter is $\iota = 0.9$, implying prices adjust on average every 10 quarters. This relatively high degree of price stickiness compromises between micro evidence on the frequency of price changes and macro evidence for a flat Phillips curve. Harding et al. (2022) show this trade-off results

[17] Other studies have considered policy rules in which the inertial term is on the Taylor-rule implied rate, $R_{T,t}$, rather than the actual policy rate, R_t. To the extent that such a rule is credible ($R_{T,t}$ is a latent variable), it also increases the effectiveness of monetary policy in a standard ZLB scenario. Thus, this latter formulation is more akin to explicit forward guidance, whereas in our specification inertia is a structural feature of monetary policy that is orthogonal to whether the economy is at the ZLB or not.

from using a CES rather than a Kimball aggregator as in Smets and Wouters (2007). Finally, bankers' survival probability is $\theta = 0.975$, implying an average tenure of 10 years.

Block B parameters are calibrated to match steady state values with long-run averages in the data. The utility weight on labor is $\chi = 3.411$ to normalize steady state labor supply to 1/3. Based on US financial balance sheet and interest rate data (see Appendix B.3), the two financial sector parameters, $\lambda = 0.411$ and $\omega = 0.001$, are calibrated to match a steady state leverage ratio of 4 and a credit spread, $400(\frac{R_k}{R_d} - 1)$, of 1%. The reserve ratio is $\alpha = 0.2$, in line with the post-financial crisis average for both the euro area and US.

Table 1: Structural parameter values

Block A. Standard parameters					
β	Discount factor	0.990	$\hbar$	Habit parameter	0.815
φ	Inverse Frisch elasticity	0.276	γ	Capital share	0.330
δ	Depreciation rate	0.025	ϵ	Elasticity of substitution	4.167
ι	Probability of fixed prices	0.900	θ	Survival probability of bankers	0.975
ϕ_π	Policy rule inflation response	1.500	ϕ_x	Policy rule output response	0.125
ρ_ζ	Persistence of risk premium shocks	0.800	ρ_ϵ	Persistence of cost-push shocks	0.800
Block B. Steady state calibrated parameters					
χ	Utility weight on labor	3.411	α	Reserve-to-deposit ratio	0.200
λ	Fraction of divertible assets	0.411	ω	Transfer to new bankers	0.001
Block C. Estimated parameters					
η	Inverse investment elasticity	1.617	ρ	Policy rule inertia	0.856
σ_ζ	S.d. of risk premium innovations	0.002	σ_ϵ	S.d. of cost-push innovations	0.033

Block C parameters are estimated using the simulated method of moments.[18] We target ten US time-series moments and five yield curve moments to estimate four parameters $\Theta = \{\eta, \rho, \sigma_\zeta, \sigma_\epsilon\}$, the inverse investment elasticity parameter, the policy rule inertia coefficient, and the standard deviations of risk premium and cost-push innovations. We estimate the inverse investment elasticity ($\eta = 1.617$) as its value is not well-informed by the literature and it determines the financial accelerator and the interest margin channel. We also estimate policy inertia ($\rho = 0.856$) because it is key for the signalling channel. The estimation suggests a significant amount of policy smoothing. Appendix B.4 shows this aligns with the existing literature and suggestive negative rates evidence from Sweden.

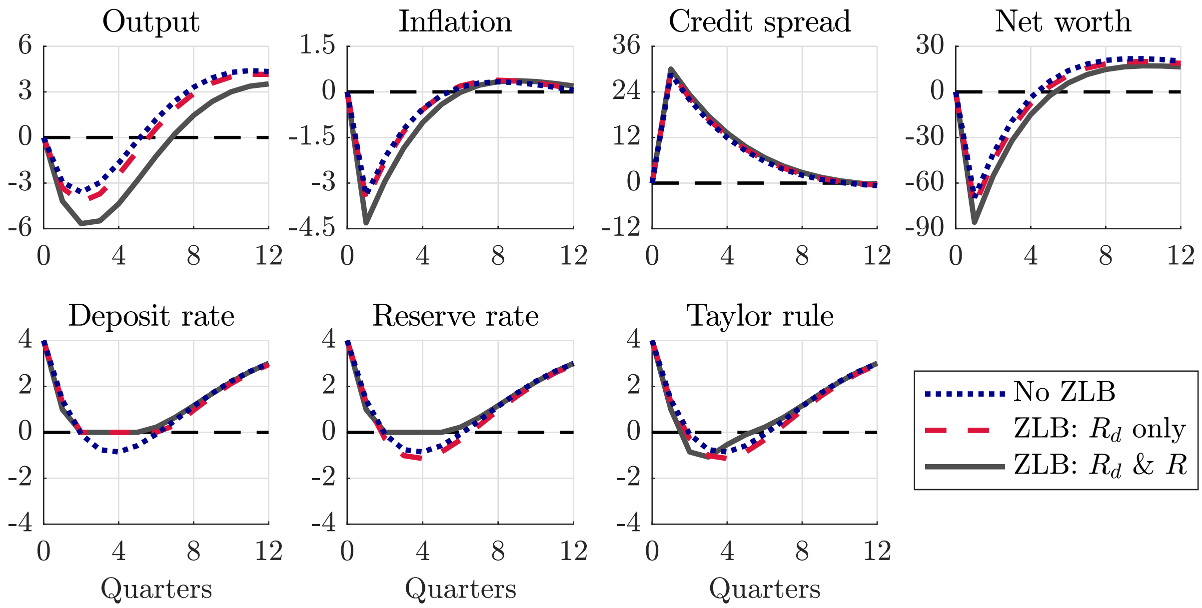
[18] Appendix B.3 documents the data sources and transformations, estimation methodology, and results.

3.3 Main results

This section presents our main results on the effectiveness of negative interest rates and illustrates the transmission mechanism using a novel decomposition of bank profits.

Effectiveness of negative rates Our baseline crisis experiment is a risk premium shock that drives the economy to the ZLB for 4 quarters.[19] Figure 4 shows the impulse responses for our policy scenarios. In all three, households cut consumption, bank net worth declines, and investment demand falls. Like the natural rate shock in Section 2, the risk premium shock shifts aggregate demand, depressing both output and inflation.[20]

Figure 4: Risk premium shock with inertia in the policy rule



Note: $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a risk premium shock that brings the economy to the ZLB for 4 quarters. All interest rates displayed are in annualized percent. All other variables are in 100×log-deviation from steady state. Inflation is annualized. Log-deviations are a good approximation of percent deviations when the deviation is small. For net worth, the −80 log-deviation, however, translates to a more modest 55 percent drop.

The smaller fall in output and inflation in the deposit rate-only ZLB scenario (II, red-dash) as compared to the standard ZLB scenario with both rates constrained (III, black-solid) indicates that negative rates—in our baseline parameterization—are expansionary. Unsurprisingly, the unconstrained scenario (I, blue-dot), where deposit rates can turn negative to offset the fall in aggregate demand, results in the smallest fall in output. When policy is constrained by the ZLB, the fall in output is largest. However, when

[19]The size of the shock is calibrated to deliver 4 periods at the ZLB in the standard ZLB scenario (III). In using a single large shock, we are trading off realism for expositional clarity.

[20]For comparison, Appendix A.3 replicates Figure 4 with the stylized model closed with the Taylor rule.

the central bank can decrease the policy rate into negative territory—despite the deposit rate being bounded by zero—it is able to extend the ZLB duration by 1 quarter and lower the post-ZLB deposit rate path providing additional stimulus.[21] Thus, a negative rate policy is expansionary even when the current deposit rate, which is relevant for households' intertemporal substitution decision, is constrained. In terms of welfare, the steady state consumption loss for the representative household to be indifferent between the unconstrained and the ZLB scenario is 0.185%, but only 0.051% with negative rates. Negative rates are expansionary (in terms of output and inflation) and welfare improving.

To isolate the quantitative response to negative interest rates in crisis times, we introduce an extra −25bp iid monetary policy shock in period 2 when the economy is at the ZLB. Figure 5 reports the impulse responses to the policy shock stripping out the effect of the underlying risk premium shock. When both the deposit and reserve rates are constrained by zero, a shock to the Taylor-rule implied rate has no effect on equilibrium outcomes (III, black-solid). Figure 5(a) shows this is not the case when the reserve rate can turn negative (II, red-dash). The policy shock becomes expansionary with peak output and inflation responses of 51% and 67% of an unconstrained shock (I, blue-dot), respectively. The deposit rate path is key to understanding this. While being constrained from period 2 (when the shock hits) until period 7, it then drops and stays persistently lower thereafter.

To explicitly identify the role of the signalling and costly interest margin channels, Figure 5(b) removes policy inertia and re-runs the previous experiment. Crucially, under the deposit rate-only ZLB scenario (II, red-dash), negative rates are now contractionary rather than expansionary, resulting in a fall in output and inflation. There are two reasons for this. First, by setting $\rho = 0$ we have switched off the signalling channel and so the fall in the reserve rate has no effect on the path of the deposit rate. Second, the costly interest margin channel results in bank net worth falling, tightening banks' incentive constraints, and causing credit spreads to rise. With the deposit rate constrained, a rise in credit spreads implies higher lending rates for firms which depresses investment demand.

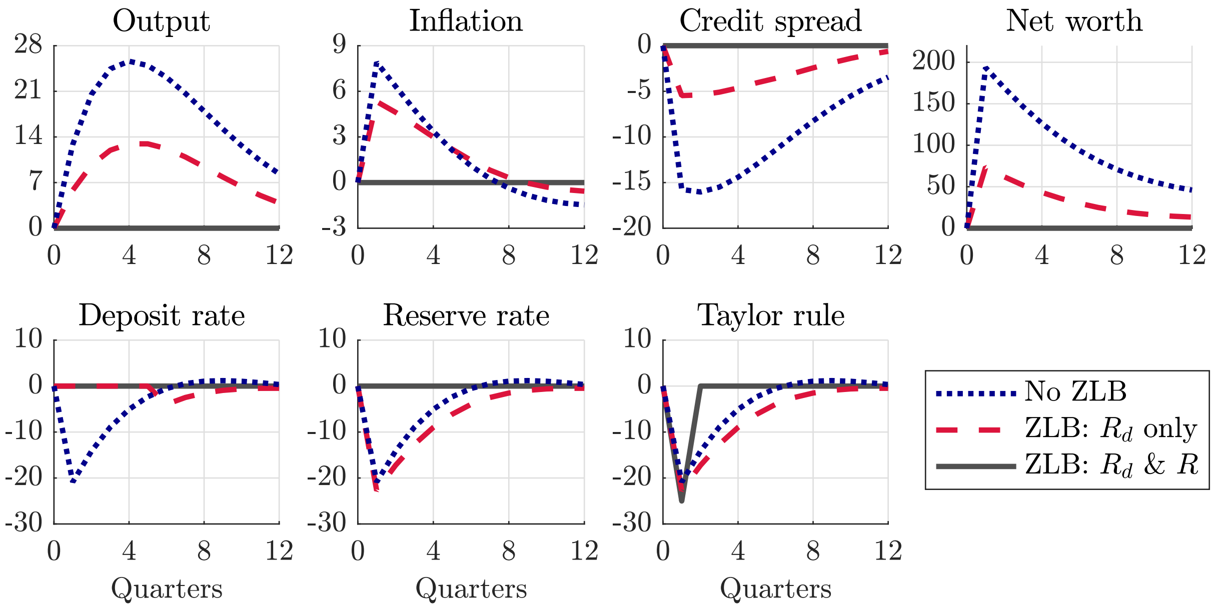
Figure 6 combines these results, decomposing the baseline deposit rate-only ZLB scenario into the signalling (blue-dot) and costly interest margin channel (red-dash). The 13bp peak output response is decomposed into a 16bp expansionary signalling channel and a −3bp contractionary interest margin channel contribution. The 5bp peak inflation response is almost completely explained by the signalling channel.[22]

[21] The effectiveness of negative rates largely appears in the output response rather than inflation, reflecting the flatness of the Phillips curve. Credit spreads are mostly driven by the exogenous risk premium shock, which is why the responses are very similar across scenarios. The lower-for-longer effect in interest rates mirrors the empirical evidence for Sweden presented in Appendix B.4.

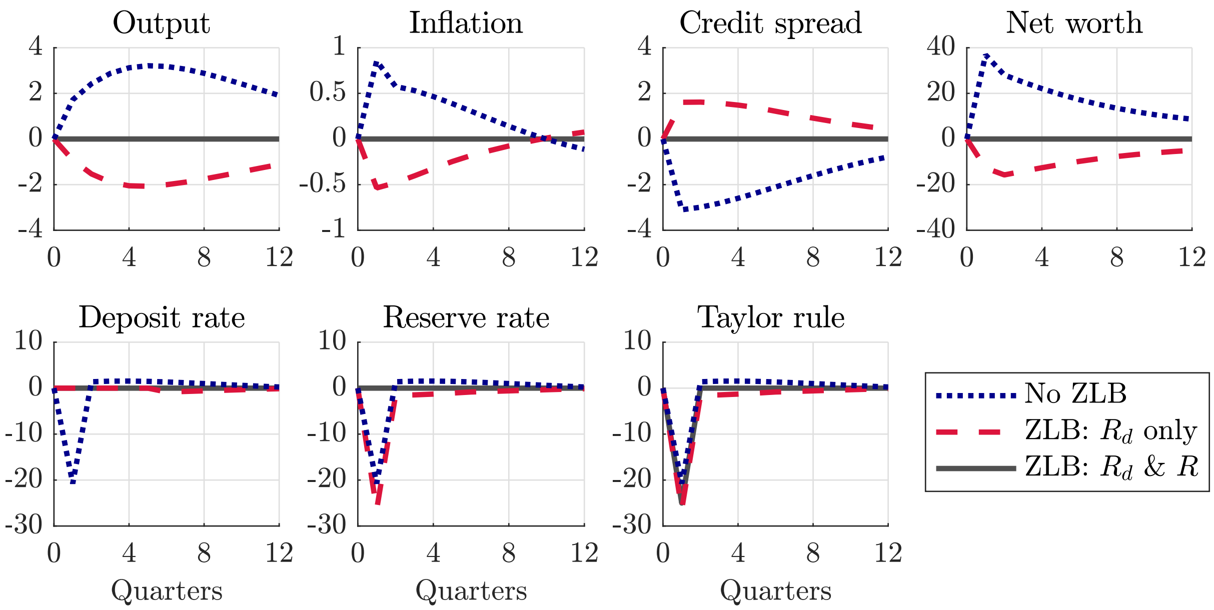
[22] Appendix B.5 replicates this decomposition for $\rho = 0$, showing that the signalling channel is indeed inactive without policy inertia. This is in a model with many endogenous state variables, which provides further evidence that "not any endogenous state variable will do" in order to generate a signalling channel.

Figure 5: Monetary policy shock in negative territory

(a) Policy rule with inertia

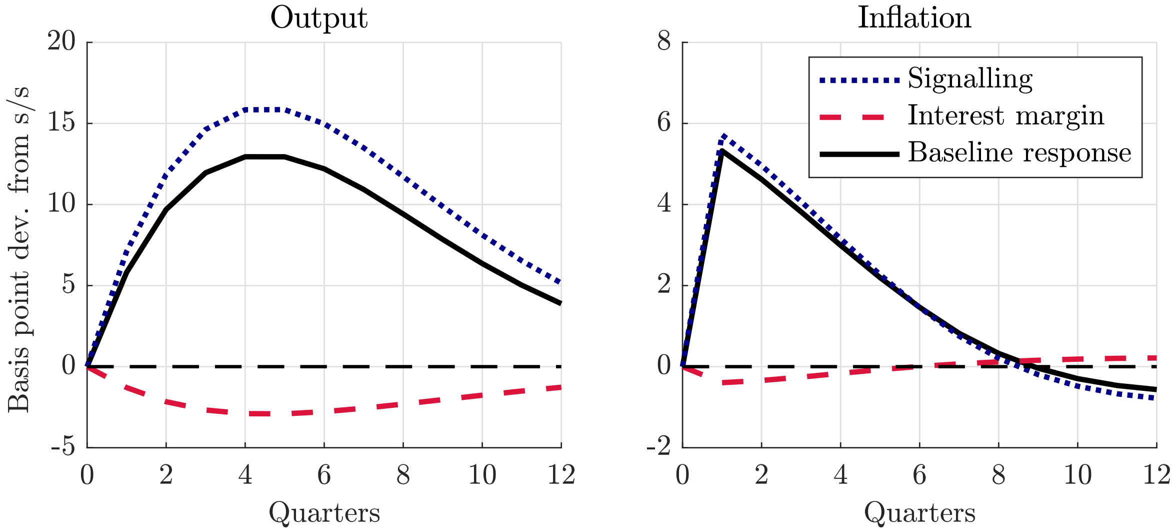


(b) Policy rule without inertia



Note: (a) $\alpha = 0.2$ and $\rho = 0.85$, (b) $\alpha = 0.2$ and $\rho = 0$. Impulse responses to a -25bp iid monetary policy shock at the ZLB. All interest rates displayed are in annualized basis points. All other variables are in basis point deviation from steady state. Inflation is annualized.

Figure 6: Contribution of signalling and interest margin channels



NOTE: Impulse responses to a −25bp iid monetary policy shock at the ZLB. Inflation is annualized. We linearly decompose the baseline response into "Signalling"—$\alpha = 0$ and $\rho = 0.85$, i.e. no costly interest margin channel—and "Interest margin"—difference between the baseline and "Signalling".

Decomposition of bank profits Bank net worth is key for the transmission of negative rates. We explore this with a novel decomposition of bank profits, $\texttt{prof}_t$, defined as the gross growth rate of a nominal net worth, conditional on not exiting. Its log-linear form can be decomposed into 3 windfall (or "surprise") and 4 predetermined terms given by

$$\hat{\texttt{prof}}_t = \underbrace{\frac{R_k \Phi}{\texttt{prof}} \left(\hat{\pi}_t - \mathbb{E}_{t-1}\hat{\pi}_t\right)}_{\text{Surprise: Inflation}} + \underbrace{\frac{\texttt{mpk}\Phi}{\texttt{prof}} \left(\hat{\texttt{mpk}}_t - \mathbb{E}_{t-1}\hat{\texttt{mpk}}_t\right)}_{\text{Surprise: Dividend}} + \underbrace{\frac{\Phi}{\texttt{prof}} \left(\hat{q}_t - \mathbb{E}_{t-1}\hat{q}_t\right)}_{\text{Surprise: Capital gain}}$$
$$+ \underbrace{\frac{\texttt{cs}\Phi}{\texttt{prof}} \hat{\texttt{cs}}_{t-1}}_{\text{Credit spread}} + \underbrace{\frac{\texttt{cs}\Phi}{\texttt{prof}} \hat{\phi}_{t-1}}_{\text{Leverage}} + \underbrace{\frac{R_d}{\texttt{prof}} \hat{r}_{d,t-1}}_{\text{Deposit rate}} - \underbrace{\frac{\alpha}{1-\alpha} \frac{R_d\left(\Phi - 1\right)}{\texttt{prof}} \left(\hat{r}_{d,t-1} - \hat{r}_{t-1}\right)}_{\text{Interest margin channel}}, \quad (30)$$

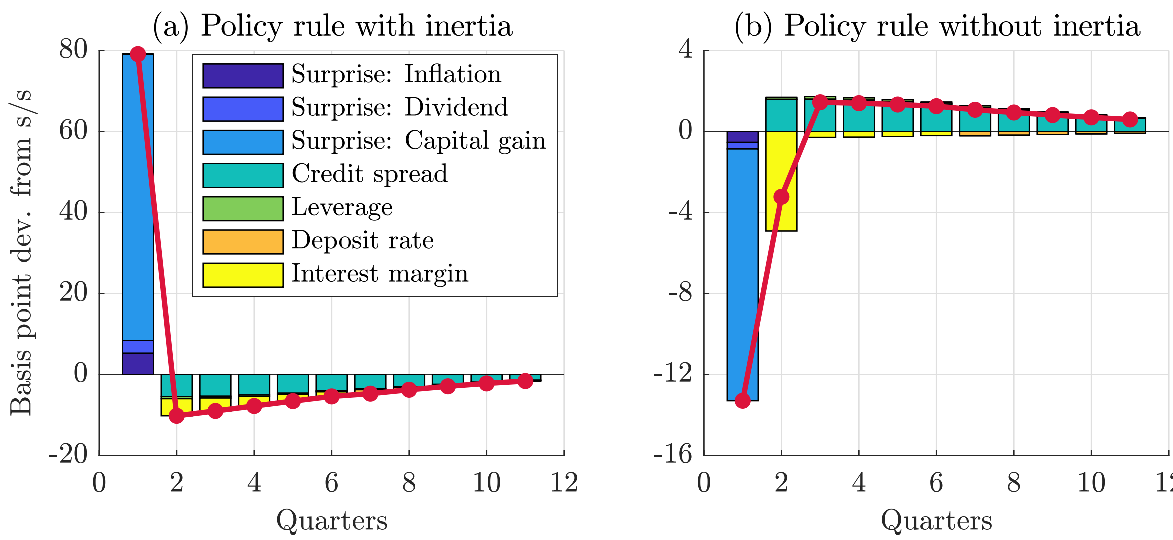
where hats denote log-deviations from steady state, variables without subscripts are steady states, $\texttt{cs}_t \equiv \mathbb{E}_t \Pi_{t+1} R_{k,t+1} - R_{d,t}$ is the nominal credit spread, and $\texttt{mpk}_t \equiv P_{m,t}\gamma Y_t / K_{t-1}$ is the marginal product of capital.[23] In general, the return on any asset can be split into a dividend payout and a capital gain. Accordingly, for banks' assets, we term the surprise change in the marginal product of capital the "dividend" and the leveraged surprise change in the asset price the "capital gain". The third windfall term is surprise inflation since we decompose nominal profits. The four predetermined terms are the evolution of i) the credit spread, ii) leverage, iii) the risk-free rate, and iv) the partial equilibrium effect of negative rates on interest margins (i.e. the costly interest marginal channel).

[23] Appendix B.6 derives the decomposition, both for the baseline model and an extended version with firm equity and loan finance introduced in Section 3.4 below.

Figure 7 plots the decomposition of bank profits in response to a −25bp iid monetary policy shock at the ZLB. In Panel (a), with the signalling channel switched on, on impact, profits sharply increase due, almost entirely, to the surprise capital gain—i.e., a revaluation of the banks' assets in response to the monetary easing. With the signalling channel operative, a lowering of the reserve rate into negative territory depresses the future expected deposit rate path. Households bring forward consumption causing aggregate production and the price of capital to increase instantaneously, driving up bank profits. From period 2 on, tighter credit spreads (the revaluation of bank assets raises net worth, slackens the banks' incentive compatibility constraint, contracting credit spreads) and the costly interest margin channel reduce profits to bring net worth back to steady state. This decomposition of how negative rates affect different parts of banks' balance sheets is consistent with empirical evidence in, for example, Altavilla et al. (2018).

In Panel (b), with signalling switched off, bank profits fall in response to the negative rate shock. Without policy inertia, negative rates come without an expansionary aggregate demand effect but solely reduce net worth via the costly interest margin channel. Lower net worth implies rising credit spreads that affect the profit decomposition in two ways. One, on impact, higher expected credit spreads depress firms' investment demand and induce capital losses. Two, from period 2 on, higher realized credit spreads generate additional profits that bring net worth back to steady state. Compared to Panel (a), all but one partial equilibrium term switch sign—the only consistently contractionary term is the interest margin channel which reduces bank profits irrespective of ρ.

Figure 7: Decomposition of bank profits



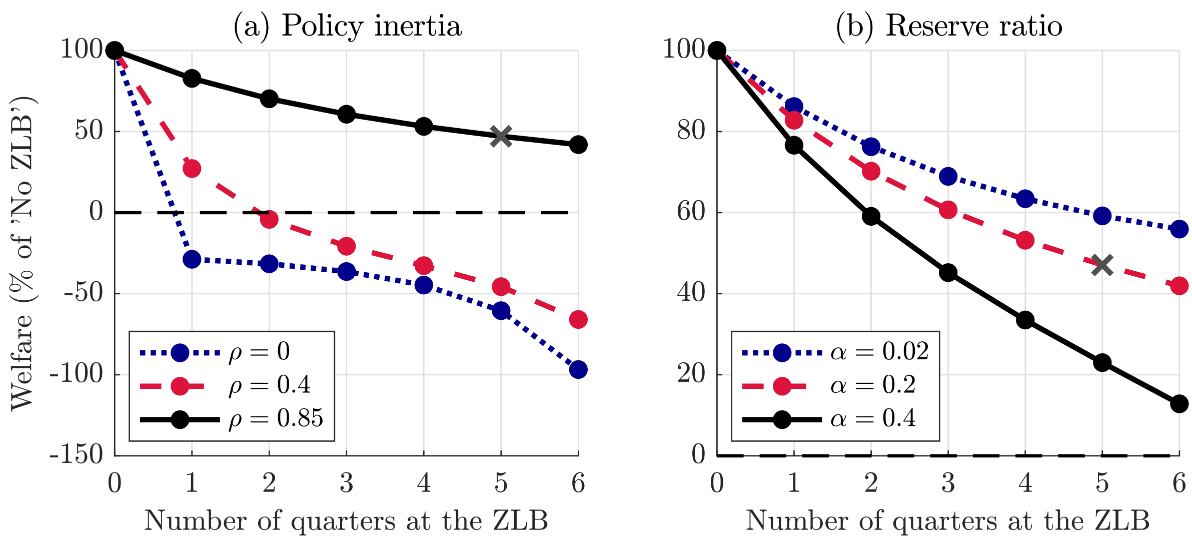
NOTE: (a) $\alpha = 0.2$ and $\rho = 0.85$, (b) $\alpha = 0.2$ and $\rho = 0$. The red-dot line plots the impulse response of bank profits to a −25bp iid monetary policy shock at the ZLB. Stacked bars decompose the impulse response for every period.

3.4 Sensitivity, forward guidance puzzle, and equity vs loans

The results above show negative rates can be both expansionary and contractionary. This section investigates more thoroughly the factors that determine their effectiveness.

Sensitivity analysis Figure 8 plots the welfare gain from a -25bp iid monetary policy shock at the ZLB for different values of policy inertia, ρ, banks' reserve-to-deposit ratio, α, and the ZLB duration. The x-axis plots the number of quarters the ZLB is expected to bind when the monetary policy shock is introduced. It scales with the size of the risk premium shock and proxies the severity of the crisis. The y-axis is the welfare gain from the monetary policy shocks as a percentage of the welfare gain from an unconstrained monetary policy shock. When the ZLB binds for zero quarters (the model is unconstrained) the value reported is 100%. This normalization ensures we strip out the effect of parameter changes on the effectiveness of "conventional" monetary policy in the model.[24]

Figure 8: Policy inertia, reserve ratio, and ZLB duration



NOTE: The x-axis scales with the size of the initial risk premium shock. The y-axis reports welfare in consumption equivalent units in response to a -25bp iid monetary policy shock for the corresponding ZLB duration relative to the welfare effect of an unconstrained monetary policy shock. The $\times$ denotes the baseline experiment.

[24]Figures 5 illustrates the need for this. The peak output response to an unconstrained monetary policy shock (I, blue-dot) is 26bps when $\rho = 0.85$ but only 3bps when $\rho = 0$. To remove this effect in our sensitivity analysis, we report results relative to unconstrained policy with the same parameter values.

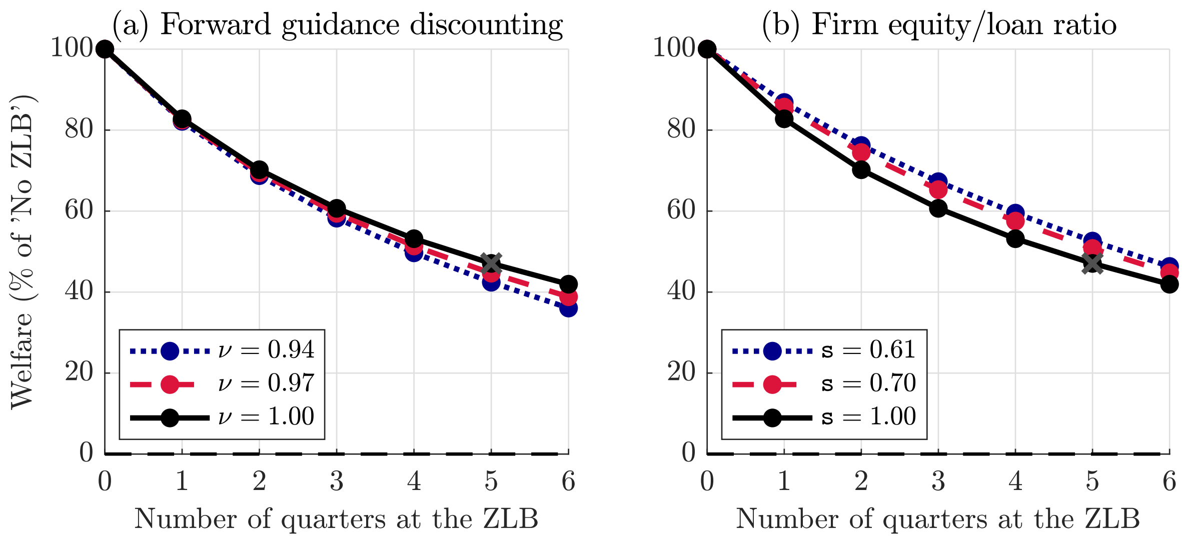
The figure shows negative rates are less effective if the ZLB is expected to bind for longer. If the deposit rate is constrained for a long period, the effect of reducing the reserve rate today only marginally lowers the future expected deposit rate path. By increasing the crisis severity (from 5 to 6 quarters at the ZLB), the welfare gain of negative rates drops from 47% to 42% of an unconstrained policy easing when $\rho = 0.85$. Panel (a) also shows negative rates are less effective for a central bank with a lower degree of policy inertia. For example, with an inertia of $\rho = 0.4$, negative rates are only welfare improving in a 1-period ZLB scenario. Panel (b) shows that when banks hold a larger reserve ratio the effectiveness of negative rates diminishes as the costly interest margin channel is stronger. For example, doubling reserve holdings to an extreme value of $\alpha = 0.4$ results in negative rates being only marginally welfare improving in a 6-period ZLB scenario. In this case, the signalling channel only just dominates the interest margin channel.

Overall though, our main result that negative rates are an effective policy tool is fairly robust. Even with $\rho = 0.8$ (the lowest degree of policy inertia documented in Appendix B.4), $\alpha = 0.27$ (the largest reserve ratio documented in Appendix B.3), and a severe crisis with 6 periods at the ZLB, a negative rate policy is still welfare improving in our model.

Forward guidance puzzle One criticism of the new-Keynesian paradigm is that equilibrium outcomes are too sensitive to future interest rate changes (Del Negro et al., 2012). Following McKay et al. (2017), we resolve the forward guidance puzzle with additional discounting, $\nu \leq 1$, in the consumption Euler equation, dampening households' sensitivity to expected future interest rate changes. The augmented Euler equation can be written as $1 = \mathbb{E}_t \beta \frac{\mu_{t+1}^{\nu}}{\mu^{\nu-1}\mu_t} \frac{\exp(\zeta_t) R_{d,t}}{\Pi_{t+1}}$, where μ_t is the marginal utility of consumption and the additional discounting is introduced in such a way so as not to distort the steady state. A first-order approximation yields the same equation as in McKay et al. (2017).

Figure 9(a) shows the signalling channel is both qualitatively and quantitatively robust to the introduction of discounting. The differences between our baseline model ($\nu = 1$), the value in McKay et al. (2017) ($\nu = .97$), and a more extreme version ($\nu = .94$) are small. For example, in our crisis scenario, dampening forward guidance only reduces the relative welfare gain from negative rates from 47% to 45% and 42%, respectively.

Figure 9: Forward guidance discounting, firm equity/loan ratio, and ZLB duration



Note: The x-axis scales with the size of the initial risk premium shock. The y-axis reports welfare in consumption equivalent units in response to a −25bp iid monetary policy shock for the corresponding ZLB duration relative to the welfare effect of an unconstrained monetary policy shock. The × denotes the baseline experiment.

Equity vs loan finance In common with Gertler and Karadi (2011), banks provide external finance to firms by purchasing their equity in our model. This gives rise to a stochastic return on bank assets and windfall dividends and capital gains in the profit decomposition (Figure 7). In reality, a large share of firms' external finance is loans. We therefore augment our model by making a fraction of banks' assets loans that earn a predetermined return. This reduces the role of capital gains in the transmission of negative rates but it does not imply negative rates become unattractive. Figure 9(b) plots results for two equity-to-loan ratios, $\mathtt{s} = .61$ (euro area data) and $\mathtt{s} = .70$ (US), respectively, and shows our results regarding the effectiveness of negative rates remain robust. If anything, negative rates are relatively more effective than in our baseline parameterization ($\mathtt{s} = 1$).[25]

In summary, since we report the effectiveness of negative rates normalized relative to the effectiveness of unconstrained policy, modifications of the model, such as ν or $\mathtt{s}$, only affect our main results in so far as they have differential effects on monetary policy in negative territory and in normal times. In contrast, changes to ρ, α, and the ZLB duration do affect our conclusions regarding the effectiveness of negative rates as they directly determine the strength of the signalling and costly interest margin channels.

[25] Appendix B.6 derives the augmented model and reproduces Figure 7-type profit decompositions for $\mathtt{s} = 0.61$ and 0.70. In the limit, with only loans, windfall capital gains (and dividends) are zero.

Further robustness We document four more robustness exercises in Appendix B.7. First is the Frisch labor supply elasticity. In the absence of wage rigidities, our baseline parameterization contains a relatively high labor supply elasticity. Lowering it to unity —a common compromise between micro and macro estimates as in Hazell et al. (2022)— decreases output and inflation responses to monetary policy but does little to alter the relative effectiveness of a negative rates. Second is the Phillips curve slope. Targeting an empirically realistic unemployment-inflation trade-off of 0.0062 as in Hazell et al. (2022), we simultaneously change the Frisch elasticity and Calvo parameter to generate a steeper Phillips curve slope of 0.023—the upper bound in Harding et al. (2022). This impacts the co-movement of output and inflation but not the relative effectiveness of negative rates.

Third is the investment elasticity since it largely determines the financial accelerator. In our baseline parameterization, net worth falls 194bp on impact in response to an unconstrained 25bp monetary policy shock, a little below the 210bp response estimated by Jarociński and Karadi (2020), suggesting our estimated elasticity is marginally too high.[26] Decreasing the elasticity weakens the impact but increases the persistence of responses to monetary policy. In terms of bank profitability, it increases windfall capital gains (as asset prices are more responsive) but lowers windfall dividends (as investment is less responsive) to negative rates. Fourth, we augment the model with nominal wage rigidity. This allows a lower Frisch elasticity while preserving the baseline effects of monetary policy.

4 Conclusion

While QE has become a relatively standard policy tool, negative rate policies remain controversial and less well understood, adopted only by a few central banks. We highlight the signalling channel of negative interest rates and show it can dominate the costly interest margin channel—exemplifying the importance of general equilibrium effects and cautioning against partial equilibrium policy evaluation. Many commercial banks have criticized the contractionary effects of negative rates on their interest margins and profits. However, we show signalling has positive general equilibrium effects on banks' asset values and balance sheet health that are not easily attributable to negative rates.

Since our quantitative results rely on a non-optimized inertial policy rule, we also study optimal policy. Abstracting from monopolistic competition among banks, we first prove negative rates are redundant under the extreme assumption of full commitment. However, under more realistic conditions in which policymakers cannot fully commit but have a preference for policy smoothing, we show negative rates can be welfare improving.

[26]As we do not target this moment in our estimation though, we take the close fit of model-implied and empirical impulse responses as supportive external validation of our baseline parameterization.

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— Appendix —

The Signalling Channel of Negative Interest Rates

Oliver de Groot and Alexander Haas | 01 May 2023

A Stylized model and optimal policy

Appendix A relates to Section 2 on optimal policy in the stylized model. Section A.1 derives a simple model of reserve demand. Section A.2 documents the full derivation of the stylized model. Section A.3 shows that the stylized model captures key features of the quantitative model if a Taylor-type policy rule is added. Section A.4 derives the first-order conditions under commitment and discretion and proves Propositions 2 and 3, respectively. Section A.5 shows that not any private-sector state variable makes negative rates optimal. Section A.6 describes the non-linear solution algorithm used to generate our numerical results. Section A.7 derives the consumption equivalent measure of welfare and provides welfare results. Section A.8 documents one additional optimal policy experiment. Finally, Section A.9 derives the analytical solutions for a simplified version of the model used for comparative statics in the main text.[27]

[27] For expositional clarity, we simplify the notation compared to Section 2. In particular, we drop time subscripts and replace them with recursive notation. y denotes the output gap.

A.1 Set up: A simple model of reserve demand [Section 2.1]

We analyse a single bank's reserve demand decision. At $t = 0$, the bank has L loans and $D_r = L$ retail deposits (without loss of generality, we set $D_r = 1$). The bank also raises wholesale deposits, $D_w \geq 0$, and places them in its reserve account at the central bank to obtain $A = D_w$ reserves. At $t = 1$, loans are repaid at R_l, reserves are repaid at R, and all deposits $(D_r + D_w)$ are repaid at R_d. At $t = 2$, a fraction $\sigma \in (0, 1)$ of total deposits ($\tilde{D} = \sigma(1 + D_w)$) flow out of the bank with probability 1/2. The cost function $\frac{2\theta}{1+\xi}\left(\max\left[\tilde{D} - A, 0\right]\right)^{1+\xi}$ (for $\xi > 1$) captures interbank market frictions and the illiquidity of loans (reserves are perfectly liquid). The bank solves the following problem:

$$\max_A \mathbb{E}\left\{(R_l - R_d) + (R - R_d)A - \frac{2\theta}{1+\xi}\left(\max\left[\tilde{D} - A, 0\right]\right)^{1+\xi}\right\},$$

$$\max_A \left\{(R_l - R_d) + (R - R_d)A - \frac{\theta}{1+\xi}\left(\max\left[\sigma(1 + A) - A, 0\right]\right)^{1+\xi}\right\}. \tag{A1}$$

The solution is as follows: If $R > R_d$, the demand for reserves is unbounded. If $R < R_d$, the bank will optimally chose a level of reserves such that a potential outflow of deposits is associated with non-zero cost (the left-hand side of the max operator). In this case, the optimal level of reserves, A^*, is given by

$$A^* = \frac{\sigma}{1-\sigma} - \frac{1}{1-\sigma}\left(\frac{R_d - R}{\theta(1-\sigma)}\right)^{1/\xi}. \tag{A2}$$

Optimal reserve holdings, A^*, are increasing in the level of liquidity risk, σ. When there is no liquidity risk, $\sigma = 0$, the bank holds no reserves. Optimal reserve holdings are also increasing in the illiquidity of loans, θ. When loans are fully liquid, $\theta = 0$, the bank holds no reserves. Defining $x \equiv R/R_d$ and the reserve-to-deposit ratio as $\alpha \equiv A/(1 + A)$, we can rewrite the demand curve, $\alpha(x)$, as

$$\alpha(x) = \frac{\sigma - \left(\frac{R_d(1-x)}{\theta(1-\sigma)}\right)^{1/\xi}}{1 - \left(\frac{R_d(1-x)}{\theta(1-\sigma)}\right)^{1/\xi}}. \tag{A3}$$

This demand curve has the following properties: $\alpha(1) = \sigma > 0$, $\alpha'(x) > 0$, and $\alpha''(x) > 0$.

A.2 Log-linear equilibrium: derivation [Section 2.2]

New-Keynesian IS equation The household problems and first-order conditions are given in the main text. In steady state, $R_d = 1/\beta$. The log-linear form of the first-order conditions for the saver household are given by

$$c_{s,t} = \mathbb{E}_t c_{s,t+1} - \frac{1}{\sigma}\left(r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t\right), \tag{A4}$$

$$\varphi l_{s,t} = -\sigma c_{s,t} + w_{s,t}, \tag{A5}$$

where lower case letters refer to log-levels. The borrower household's conditions are

$$c_{b,t} = \mathbb{E}_t c_{b,t+1} - \frac{1}{\sigma}\left(r_{b,t} - \mathbb{E}_t \pi_{t+1} - s_t\right), \tag{A6}$$

$$\varphi l_{b,t} = -\sigma c_{b,t} + w_{b,t}, \tag{A7}$$

where, in steady state, $R_b = 1/\beta_b$. The log-linear aggregate resource constraint is given by $y_t = (1 - \mathsf{c})\, c_{s,t} + \mathsf{c} c_{b,t}$, where $\mathsf{c} \equiv C_b/Y$. Combining this definition with the two individual Euler equations gives the aggregate Euler equation:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1-\mathsf{c}}{\sigma}\left(r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t\right) - \frac{\mathsf{c}}{\sigma}\left(\mathbb{E}_t r_{b,t+1} - \mathbb{E}_t \pi_{t+1} - s_t\right). \tag{A8}$$

Next, substituting the transfer from savers to borrowers into the borrower household's budget constraint gives the following simple borrower household consumption function: $C_{b,t} = B_t$. Using the definition for leverage, $\Phi_t = B_t/N_t$, the log-linear form of the borrower household consumption function is given by $c_{b,t} = \phi_t + n_t$. Rearranging the borrower household's Euler condition, $\frac{1}{\sigma}\left(r_{b,t} - \mathbb{E}_t \pi_{t+1} - s_t\right) = \mathbb{E}_t c_{b,t+1} - c_{b,t}$, and combining it with the consumption function above, we can rewrite the aggregate Euler equation as

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1-\mathsf{c}}{\sigma}\left(r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t\right) - \mathsf{c}\left(\mathbb{E}_t \phi_{t+1} - \phi_t + \mathbb{E}_t n_{t+1} - n_t\right). \tag{A9}$$

New-Keynesian Phillips curve Log-linearizing the production sector's first-order conditions yields the textbook new-Keynesian Phillips curve in terms of marginal cost,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\iota\beta)(1-\iota)}{\iota} mc_t. \tag{A10}$$

Log-linear marginal cost and aggregate output are given by $mc_t = \omega w_{s,t} + (1-\omega) w_{b,t}$ and $y_t = \omega l_{s,t} + (1-\omega) l_{b,t}$, respectively. Using the two labor-supply first-order conditions from the household problem, we can rewrite marginal cost as follows:

$$mc_t = (\varphi + \sigma) y_t, \tag{A11}$$

and the Phillips curve as

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\iota\beta)(1-\iota)(\varphi+\sigma)}{\iota} y_t. \tag{A12}$$

Note that since we only consider disturbances to households' subjective discount factors, the output gap coincides with output and hence y_t can be relabeled as the output gap.

Financial sector equilibrium conditions Steady state leverage is given by $\bar{N}$. The log-linear net worth evolution equation is given by

$$n_{t+1} = \theta R \left(n_t + \Phi (r_{b,t} - \pi_{t+1}) - (\Phi - 1) \left(\frac{r_{d,t} - \alpha r_t}{1-\alpha} - \pi_{t+1} \right) \right). \tag{A13}$$

When $\theta = 0$, then $n_{t+1} = 0$. The steady state tax on banks ensures that in steady state $R_b (1-\tau) = R_d$. The log-linear incentive compatibility constraint is given by

$$\phi_t = (\mathbb{E}_t m_{t,t+1} - \pi_{t+1}) + \theta \mathbb{E}_t \phi_{t+1} + \left(\Phi r_{b,t} - (\Phi - 1) \frac{r_{d,t} - \alpha r_t}{1-\alpha} \right). \tag{A14}$$

where $m_{t,t+1}$ is the log-linear stochastic discount factor of the saver household.

Substituting for $r_{b,t}$ using the borrower household's Euler equation gives

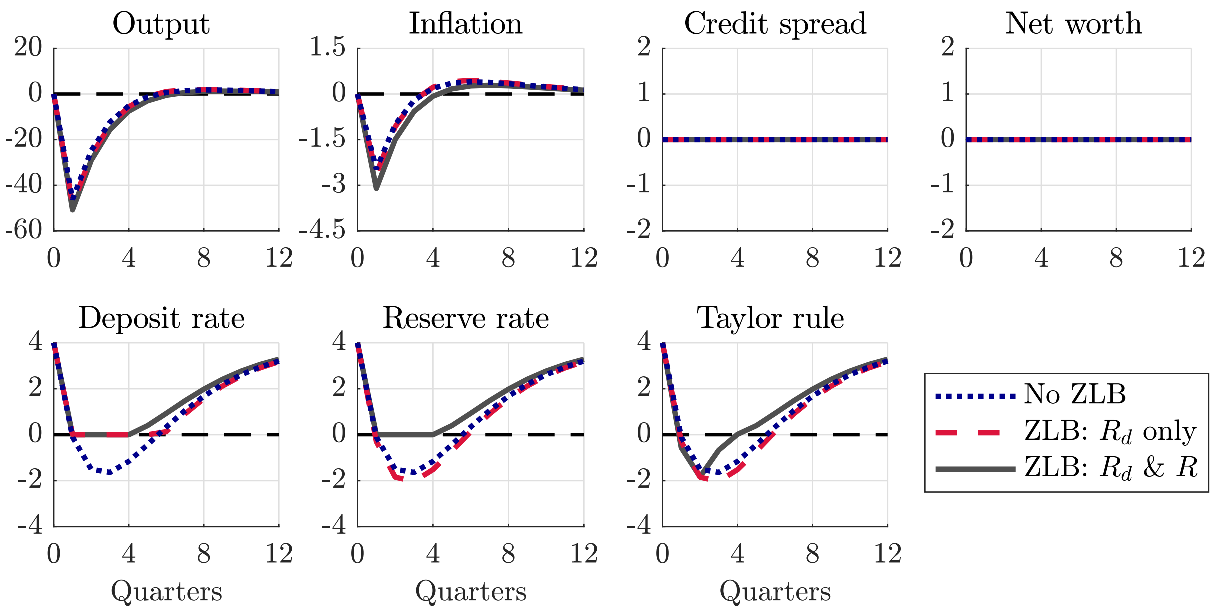
$$\begin{aligned} \phi_t = &-r_{d,t} + \theta \mathbb{E}_t \phi_{t+1} + \Phi \sigma (\mathbb{E}_t \phi_{t+1} - \phi_t + \mathbb{E}_t n_{t+1} - n_t) \\ &+ \Phi (\mathbb{E}_t \pi_{t+1} + s_t) - (\Phi - 1) \frac{r_{d,t} - \alpha r_t}{1-\alpha}. \end{aligned} \tag{A15}$$

Rearranging and setting $\theta = 0$ such that $n_t = 0$ gives Equation (15) in the main text. When $\theta > 0$, the model is described by five endogenous variables, $\{\pi_t, y_t, \phi_t, n_t, r_{d,t}\}$, and four private-sector conditions, (A9), (A12), (A13), and (A15).

A.3 Log-linear equilibrium: a Taylor-type rule [Section 2.2]

By replicating the main results from Section 3, this section shows that the stylized model captures the key features of the quantitative model. The experiments are conducted combining the IS and Phillips curve of the stylized model, Equations (13) and (16), respectively, and the Taylor-type rule of the quantitative model, (29).

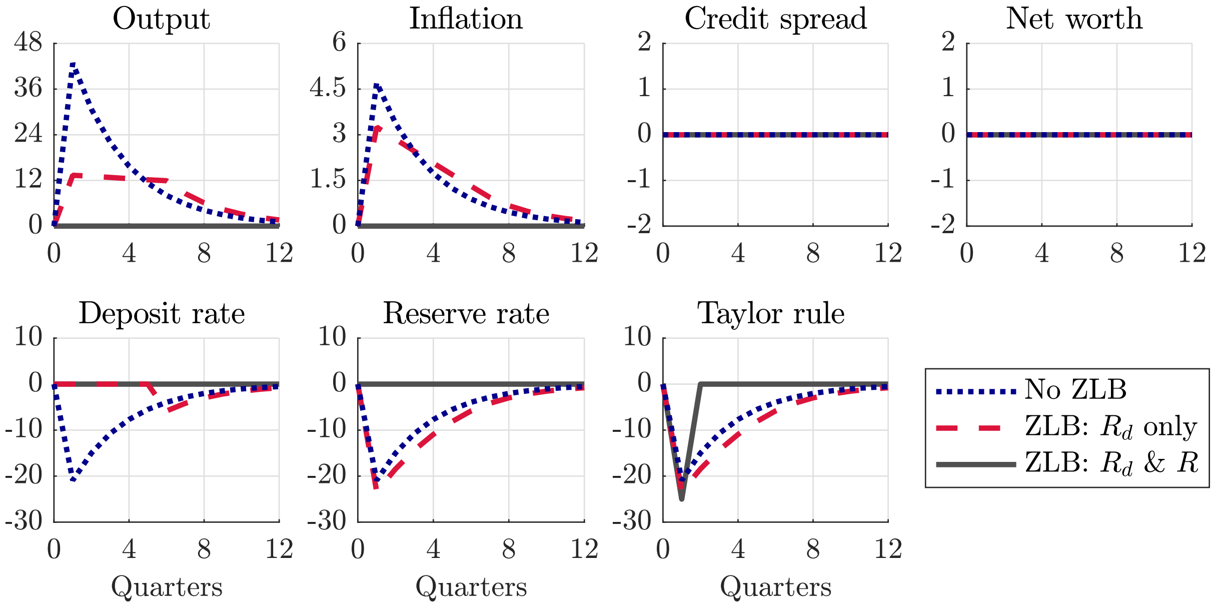
Figure A1: Stylized model: Natural rate shock with inertia in the policy rule



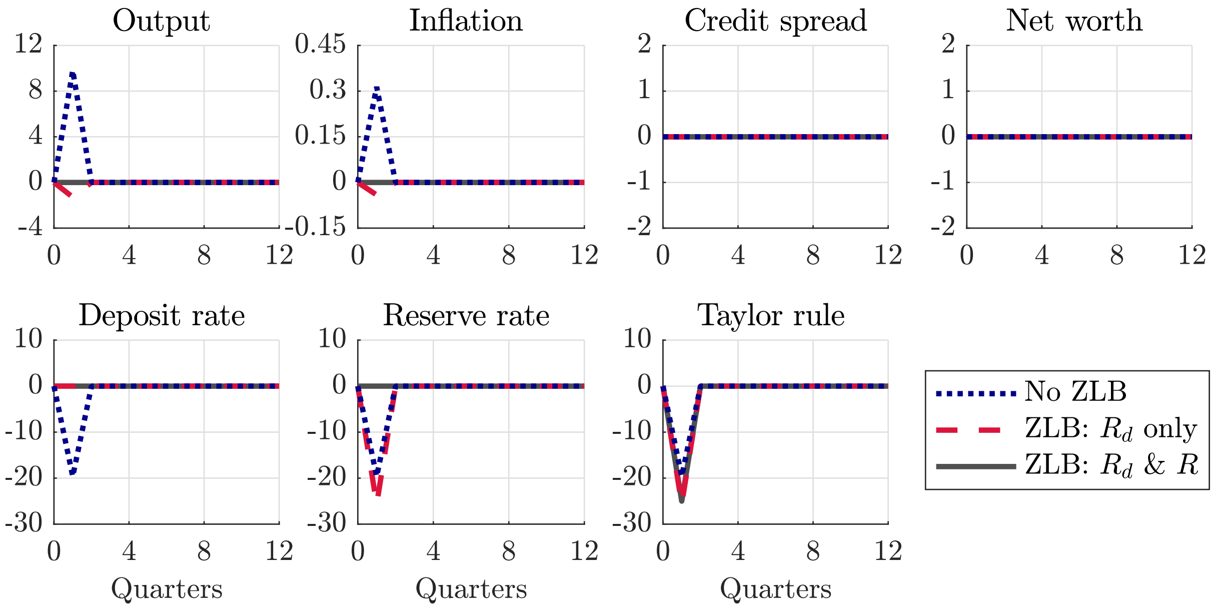
Note: Stylized model from Section 2 with a Taylor-type policy rule. $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a natural rate shock that brings the economy to the ZLB for 4 quarters. All interest rates displayed are in annualized percent. Other variables are in 100×log-deviation from steady state. Inflation is annualized.

Figure A2: Stylized model: Monetary policy shock in negative territory

(a) Policy rule with inertia

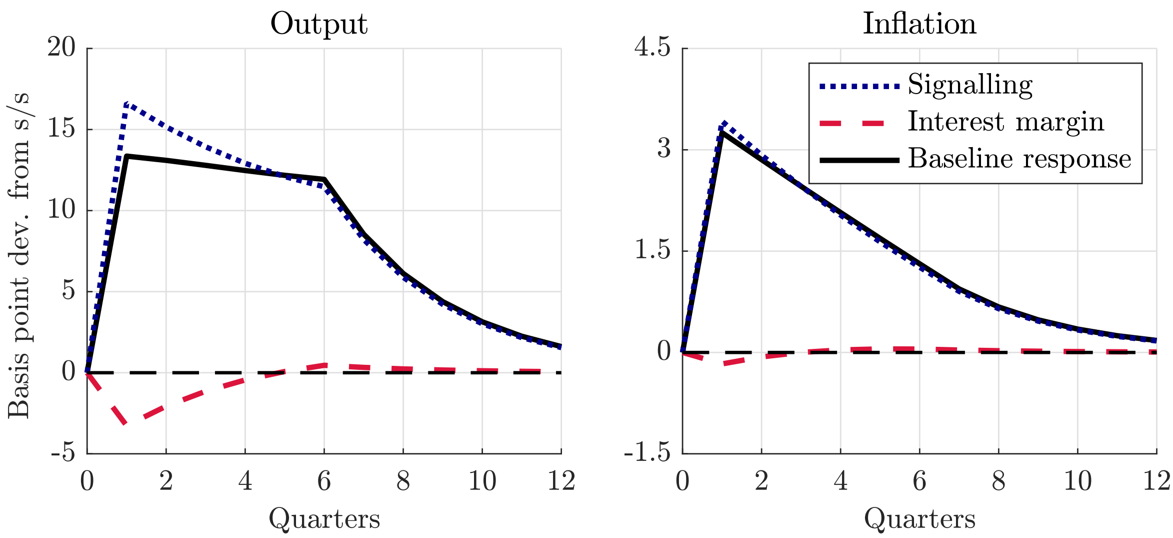


(b) Policy rule without inertia



Note: Stylized model from Section 2 with a Taylor-type policy rule. (a) $\alpha = 0.2$ and $\rho = 0.85$, (b) $\alpha = 0.2$ and $\rho = 0$. Impulse responses to a -25bp iid monetary policy shock at the ZLB. All interest rates displayed are in annualized basis points. Output and inflation are in basis point deviation from steady state. Inflation is annualized.

Figure A3: Stylized model: Contribution of signalling and interest margin channels



Note: Stylized model from Section 2 with a Taylor-type policy rule. Impulse responses to a −25bp iid monetary policy shock at the ZLB. Inflation is annualized. We linearly decompose the baseline response into "Signalling"—$\alpha = 0$ and $\rho = 0.85$, i.e. no costly interest margin channel—and "Interest margin"—difference between the baseline and "Signalling".

A.4 Analytical results: propositions 2 and 3 [Section 2.3]

The recursive problem of the optimal policymaker is given by

$$V(\pi_{-1}, s) = \max_{\{\pi,y,r_d,r\}} -\frac{1}{2}\left(\pi^2 + \lambda y^2\right) + \beta \mathbb{E} V(\pi, s_{+1})$$

$$\pi = \beta \mathbb{E}\pi_{+1} + \gamma\pi_{-1} + \kappa y, \tag{PC}$$

$$y = \mathbb{E}y_{+1} - \sigma^{-1}\left(r_d - \mathbb{E}\pi_{+1} - s\right) - \phi\left(r_d - r\right), \tag{IS}$$

$$r_d \geq 0 \quad \text{(ZLB)}, \quad r_d - r \geq 0 \quad \text{(ARB)}, \quad r_d\left(r_d - r\right) = 0 \quad \text{(X)},$$

where the decentralized competitive equilibrium and a set of three inequality constraints on the policy tools constrain the optimal choice. This model is a slightly generalized version of the stylized model in Section 2. All proofs hold with lagged inflation added to the new-Keynesian Phillips Curve—resulting from, for example, price indexation.

Under **commitment**, the equilibrium can be summarized by the following equations:

$$
\begin{aligned}
& & \pi &= \beta \mathbb{E}\pi_{+1} + \gamma\pi_{-1} + \kappa y, \\
& & y &= \mathbb{E}y_{+1} - \sigma^{-1}\left(r_d - \mathbb{E}\pi_{+1} - s\right) - \phi\left(r_d - r\right), \\
\pi :& & 0 &= \pi - \beta \mathbb{E}\boldsymbol{V_1}(\pi, s_{+1}) - \zeta_{PC} + \zeta_{PC_{-1}} + \sigma^{-1}\beta^{-1}\zeta_{IS_{-1}}, \\
y :& & 0 &= \lambda y + \kappa\zeta_{PC} - \zeta_{IS} + \beta^{-1}\zeta_{IS_{-1}}, \\
r_d :& & 0 &= \zeta_{IS}\left(\sigma^{-1} + \phi\right) + \zeta_{ZLB} + \zeta_{ARB} + \zeta_X\left(2r_d - r\right), \\
r :& & 0 &= \zeta_{IS}\phi + \zeta_{ARB} + \zeta_X r_d, \\
\text{KT}_1 :& & 0 &= \zeta_{ZLB} r_d, \\
\text{KT}_2 :& & 0 &= \zeta_{ARB}\left(r_d - r\right), \\
\text{EC} :& & \boldsymbol{V_1}(\pi_{-1}, s) &= -\gamma\zeta_{PC},
\end{aligned}
$$

where the ζ are Lagrange multipliers. Based on the set of three inequality constraints on the policy tools, the following regimes can be defined: `Regime I`: $\{r_d > 0,\ r = r_d\}$, `Regime II`: $\{r_d = 0,\ r < 0\}$, and `Regime III`: $\{r_d = 0,\ r = 0\}$.

Proof of Proposition 2 Proposition 2 states that, under commitment, the reserve rate will never be set negative. This is equivalent to stating, $r \in$ `Regime II` is not optimal. We prove this by contradiction.

For a given state vector, $\boldsymbol{s} = \left\{\pi_{-1}, \zeta_{IS_{-1}}, \zeta_{PC_{-1}}, s\right\}$, define $r^{\text{c,zlb}}(\boldsymbol{s})$ and $r_d^{\text{c,zlb}}(\boldsymbol{s})$ as the reserve and deposit rate, respectively, that are the solution to the constrained commitment problem where negative rates are not allowed, $r \in \{$`Regime I, Regime III`$\}$, and $r^{\text{c,nir}}(\boldsymbol{s})$ and $r_d^{\text{c,nir}}(\boldsymbol{s})$ as the reserve and deposit rate that solve the commitment problem where negative reserve rates are allowed, i.e. $r \in \{$`Regime I, Regime II, Regime III`$\}$.

Consider $\phi > 0$. Suppose $\exists\ \boldsymbol{s} \mid V^{\text{c,nir}}(\boldsymbol{s}) > V^{\text{c,zlb}}(\boldsymbol{s}) \longrightarrow r^{\text{c,nir}} < 0$ and $r_d^{\text{c,nir}} = 0$ (`Regime II`). Then, the equilibrium allocation for $\{\pi, y\}$ is given by (PC) and (IS), where (IS) can be reduced to $y = \mathbb{E}y_{+1} + \sigma^{-1}(\mathbb{E}\pi_{+1} + s) + \phi r^{\text{c,nir}}$. Yet, $r^{\text{c,*}} = r_d^{\text{c,*}} = -\phi\sigma r^{\text{c,nir}} > 0$ (`Regime I`) generates the same equilibrium allocation, $V^{\text{c,*}}(\boldsymbol{s}) = V^{\text{c,nir}}(\boldsymbol{s})$. However, $r^{\text{c,*}}$ and $r_d^{\text{c,*}}$ are in the space of the constrained commitment problem such that $V^{\text{c,*}}(\boldsymbol{s}) = V^{\text{c,nir}}(\boldsymbol{s}) \leq V^{\text{c,zlb}}(\boldsymbol{s})$. Thus, we have a contradiction.

Consider $\phi = 0$. The reserve rate in this case drops out of the equilibrium system that determines $\{y, \pi, r_d, \zeta_{IS}, \zeta_{PC}\}$ as $\phi(r_d - r) = 0\ \forall\ r$ in (IS). There is no role for negative interest rates. ■

To study optimal time-consistent policy with and without policy smoothing, we augment the policymaker's objective function by adding a preference for smoothing interest rates, given by ψ. This gives the following, slightly modified, recursive planner's problem:

$$
\begin{aligned}
V(r_{-1}, \pi_{-1}, s) = \max_{\{\pi, y, r_d, r\}} & -\frac{1}{2}\left((1-\psi)\left(\pi^2+\lambda y^2\right)+\psi\left(r-r_{-1}\right)^2\right)+\beta \mathbb{E} V(r, \pi, s_{+1}) \\
\pi &= \beta \mathbb{E}\pi_{+1} + \gamma \pi_{-1} + \kappa y, \qquad \text{(PC)} \\
y &= \mathbb{E}y_{+1} - \sigma^{-1}\left(r_d - \mathbb{E}\pi_{+1} - s\right) - \phi\left(r_d - r\right), \qquad \text{(IS)} \\
r_d &\geq 0 \quad \text{(ZLB)}, \quad r_d - r \geq 0 \quad \text{(ARB)}, \quad r_d\left(r_d - r\right) = 0 \quad \text{(X)}.
\end{aligned}
$$

Under **discretion**, the equilibrium can be summarized by the following equations:

$$
\begin{aligned}
& \pi = \beta \mathbb{E}\boldsymbol{\pi}\left(r, \pi, s_{+1}\right) + \gamma \pi_{-1} + \kappa y, \\
& y = \mathbb{E}\boldsymbol{y}\left(r, \pi, s_{+1}\right) - \sigma^{-1}\left(r_d - \mathbb{E}\boldsymbol{\pi}\left(r, \pi, s_{+1}\right) - s\right) - \phi\left(r_d - r\right), \\
\pi: \quad & 0 = (1-\psi)\pi - \mathbb{E}\boldsymbol{V_2}\left(r, \pi, s_{+1}\right) - \zeta_{PC}\left(1 - \beta \mathbb{E}\boldsymbol{\pi_2}\left(r, \pi, s_{+1}\right)\right) \\
& \quad + \zeta_{IS}\left(\mathbb{E}\boldsymbol{y_2}\left(r, \pi, s_{+1}\right) + \sigma^{-1}\mathbb{E}\boldsymbol{\pi_2}\left(r, \pi, s_{+1}\right)\right), \\
y: \quad & 0 = (1-\psi)\lambda y - \zeta_{IS} + \kappa \zeta_{PC}, \\
r_d: \quad & 0 = \zeta_{IS}\left(\sigma^{-1} + \phi\right) + \zeta_{ZLB} + \zeta_{ARB} + \zeta_X\left(2r_d - r\right), \\
r: \quad & 0 = \psi\left(r - r_{-1}\right) - \beta \mathbb{E}\boldsymbol{V_1}\left(r, \pi, s_{+1}\right) + \beta \mathbb{E}\boldsymbol{\pi_1}\left(r, \pi, s_{+1}\right)\zeta_{PC} \\
& \quad + \zeta_{IS}\left(\mathbb{E}\boldsymbol{y_1}\left(r, \pi, s_{+1}\right) + \sigma^{-1}\mathbb{E}\boldsymbol{\pi_1}\left(r, \pi, s_{+1}\right)\right) + \zeta_{ARB} + \zeta_X r_d, \\
\text{KT}_1: \quad & 0 = \zeta_{ZLB} r_d, \\
\text{KT}_2: \quad & 0 = \zeta_{ARB}\left(r_d - r\right), \\
\text{EC}_1: \quad & \boldsymbol{V}_1\left(r_{-1}, \pi_{-1}, S\right) = -\psi\left(r - r_{-1}\right), \\
\text{EC}_2: \quad & \boldsymbol{V}_2\left(r_{-1}, \pi_{-1}, S\right) = -\zeta_{PC}\gamma,
\end{aligned}
$$

where the ζ are Lagrange multipliers. Analogous to the commitment problem, once again the following three policy regimes can be defined: `Regime I`: $\{r_d > 0,\ r = r_d\}$, `Regime II`: $\{r_d = 0,\ r < 0\}$, and `Regime III`: $\{r_d = 0,\ r = 0\}$.

Proof of Proposition 3 Proposition 3 states that, under discretion, with $\psi = 0$, the reserve rate will never be set negative. Equivalently, $r \in$ `Regime II` is not optimal. We prove this by contradiction.

For a given state vector, $\boldsymbol{s} = \left\{r_{-1}, \pi_{-1}, \zeta_{IS_{-1}}, \zeta_{PC_{-1}}, s\right\}$, define $r^{\text{d,zlb}}(\boldsymbol{s})$ and $r_d^{\text{d,zlb}}(\boldsymbol{s})$ as the reserve and deposit rate, respectively, that are the solution to the constrained discretion problem where negative rates are not an option, $r \in \{$`Regime I, Regime III`$\}$, and $r^{\text{d,nir}}(\boldsymbol{s})$ and $r_d^{\text{d,nir}}(\boldsymbol{s})$ as the reserve and deposit rate that solve the discretion problem where negative reserve rates are allowed, i.e. $r \in \{$`Regime I, Regime II, Regime III`$\}$.

With $\psi = 0$, $\boldsymbol{V_1}(r_{-1}, \pi_{-1}, s) = 0$ and r_{-1} drops out as a state variable, i.e. expectations and allocations in the discretionary equilibrium are independent of r_{-1}. Thus, redefining $\boldsymbol{s} = \left\{\pi_{-1}, \zeta_{IS_{-1}}, \zeta_{PC_{-1}}, s\right\}$ we proceed as in the commitment case.

Consider $\phi > 0$: Suppose $\exists\ \boldsymbol{s} \mid V^{\text{d,nir}}(\boldsymbol{s}) > V^{\text{d,zlb}}(\boldsymbol{s}) \longrightarrow r^{\text{d,nir}} < 0$ and $r_d^{\text{d,nir}} = 0$ (`Regime II`). Then, the equilibrium allocation for $\{\pi, y\}$ is given by (PC) and (IS), where (IS) can be reduced to $y = \mathbb{E}\boldsymbol{y}(\pi, s_{+1}) + \sigma^{-1}(\mathbb{E}\boldsymbol{\pi}(\pi, s_{+1}) + s) + \phi r^{\text{d,nir}}$. Yet, $r^{\text{d,*}} = r_d^{\text{d,*}} = -\phi\sigma r^{\text{d,nir}} > 0$ (`Regime I`) generates the same equilibrium allocation, $V^{\text{d,*}}(\boldsymbol{s}) = V^{\text{d,nir}}(\boldsymbol{s})$. However, $r^{\text{d,*}}$ and $r_d^{\text{d,*}}$ are in the space of the constrained commitment problem such that $V^{\text{d,*}}(\boldsymbol{s}) = V^{\text{d,nir}}(\boldsymbol{s}) \leq V^{\text{d,zlb}}(\boldsymbol{s})$. Thus, we have a contradiction.

Consider $\phi = 0$: The reserve rate in this case drops out of the equilibrium system that determines $\{y, \pi, r_d, \zeta_{IS}, \zeta_{PC}\}$ as $\phi(r_d - r) = 0\ \forall\ r$ in (IS). There is no role for negative interest rates. ■

A.5 Analytical results: additional state variables [Section 2.3]

This section discusses the possibility whether either r_{t-1} or alternative private-sector state variables (e.g., π_{t-1} and y_{t-1}) appearing in the private sector equilibrium conditions can generate results akin to our signalling channel, thus removing the need to assume a policy smoothing motive. The discussion proceeds in two parts.

Part I Suppose that the IS curve and an inertial monetary policy rule are given by

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t \pi_{t+1} - s_t\right), \tag{A16}$$

$$r_t = f\left(\pi_t, y_t\right) + \rho r_{t-1}. \tag{A17}$$

This policy rule can be written as a geometric distributed lag of past inflation and output,

$$\begin{aligned}(1-\rho L)\, r_t &= f\left(\pi_t, y_t\right), \\ r_t &= \frac{\phi_\pi}{(1-\rho L)} f\left(\pi_t, y_t\right), \\ &= f\left(\pi_t, y_t\right) + \rho f\left(\pi_{t-1}, y_{t-1}\right) + \rho^2 f\left(\pi_{t-2}, y_{t-2}\right) + \cdots .\end{aligned} \tag{A18}$$

By substitution, in an unconstrained environment, the equilibrium paths of output and inflation in (A16) and (A17) must be equivalent to the following equilibrium that features an IS curve with a lag structure in inflation and output; and a policy rule without inertia,

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t \pi_{t+1} - s_t + \rho f\left(\pi_{t-1}, y_{t-1}\right) + \rho^2 f\left(\pi_{t-2}, y_{t-2}\right) + \cdots\right), \tag{A19}$$

$$r_t = f\left(\pi_t, y_t\right). \tag{A20}$$

Alternatively, the IS curve can also be written in terms of lagged policy rates as follows

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t \pi_{t+1} - s_t + \rho r_{t-1} + \rho^2 r_{t-2} + \cdots\right). \tag{A21}$$

Thus, unconstrained by the ZLB, a model with an appropriately chosen structure of state variables in the private sector equilibrium conditions can replicate the equilibrium of the model in which the only state variable comes from Taylor rule inertia. We do not attempt to provide a micro-foundation for such a structure. However, an important insight is that a single lagged inflation or output term is not sufficient to replicate the equilibrium of the model with smoothing, especially when ρ is large (e.g., in the order of 0.8).

While inflation and output follow the same equilibrium path, the equilibrium interest rate path is quite different. In particular, the effect on impact ($t = 0$) for r_t in response to an exogenous shock is of the same magnitude. However, after that the equilibrium path of the interest rate without smoothing features faster mean reversion. This implies that once the ZLB constraint is introduced, the equilibrium paths for inflation and output are no longer equivalent across models since the numbers of periods spent at ZLB are different.

Part II More generally, we can write our 3-equation new-Keynesian model—abstracting from the costly interest margin channel—as

$$0 = \mathbb{E}_t f\left(\pi_{t+1}, y_{t+1}, \pi_t, y_t, r_t^d\right) = 0, \tag{A22}$$

$$r_t = g\left(\pi_t, y_t\right) + \varepsilon_t, \tag{A23}$$

$$r_t^d = \begin{cases} r_t & \text{if} \quad r_t \geq 0 \\ 0 & \text{if} \quad r_t < 0 \end{cases}, \tag{A24}$$

where (A22) incorporates the private-sector equations of the Phillips and IS curve. To see that adding endogenous state variables $\left(\pi_{t-1}, y_{t-1}, r_{t-1}^d\right)$ to the private-sector equilibrium (whether because of inflation indexation, consumption habits, or long-term bonds, respectively) does not generate an effective signalling channel of negative interest rates, consider the following experiment: Suppose $r_0^* = 0$ and the equilibrium path is defined by $\left\{\pi_t^*, y_t^*, r_t^{d*}, r_t^*\right\}_{t=0}^{\infty}$. In this case, a monetary policy shock, $\varepsilon_0 < 0$, lowers r_0 but since r_0^d remains unchanged at 0, this leaves the rest of the equilibrium path unchanged, irrespective of the presence of additional state variables in the private-sector equations.

This ineffectiveness of negative interest rates disappears if the Taylor rule contains a smoothing term, for example, as follows: $r_t = g\left(\pi_t, y_t\right) + \rho r_{t-1} + \varepsilon_t$. In this case, suppose $r_0 = 0$ and $r_1 > 0$. The same iid monetary policy shock in period 0 leaves r_0^d unchanged. However, all else equal, this shock lowers r_1 and hence r_1^d. Since r_1^d enters the private-sector equilibrium conditions, this alters the equilibrium path $\left\{\pi_t, y_t, r_t^d, r_t\right\}_{t=0}^{\infty}$.

Finally, note that the reserve rate, r_t does not enter (A22). It is difficult to conjecture a microfoundation (e.g., because of sticky information) in which the reserve rate would enter as a state variable, r_{t-1}, without the original presence of r_t. Of course, when $\phi > 0$, r_t does enter via the costly interest margin channel term in the IS curve, $-\phi\left(r_{d,t} - r_t\right)$, but this has an unambiguously negative sign on output. This is also the case in the quantitative model in Section 3 where the presence of r_{t-1} has an unambiguous negative sign in the banks' net worth accumulation equation.

A.6 Numerical results: policy function iteration [Section 2.4]

To derive a solution to the time-consistent optimal policymaker's problem, we use a policy function iteration algorithm, solving for $\pi(r,g)$, $y(r,g)$, $r'(r,g)$, $r_d(r,g)$, $\zeta_{ZLB}(r,g)$, and $\zeta_{ARB}(r,g)$. The algorithm proceeds as follows:

1. Set N_i: number of points on the interest rate grid, N_s: number of exogenous states, ϵ: tolerance limit for convergence, $\mathtt{u}$: updating parameter. Set grid points $\{i_0, \ldots, i_{N_i}\}$. The AR(1) process for the natural rate, g, is approximated using Tauchen and Hussey (1991)'s quadrature algorithm that gives a set of grid points $\{s_0, \ldots, s_{N_s}\}$ and a transmission matrix, M.

2. Start iteration j with conjectured functions for $r'^j(r,g)$ and $\pi^j(r,g)$. The initial functions are set to $r'^0(r,g) = 1/\beta - 1$ and $\pi^0(r,g) = 0$. $\pi(r,g)$ is only defined at the nodes of the grids for the policy rate and shock, but since $r'(r,g)$ is generally not going to match node grids exactly, the function $\pi(r,g)$ is interpolated over the first argument to determine its values at $\boldsymbol{\pi}^j(r'^j(r,g), g')$. Construct expectations $\mathbb{E}\boldsymbol{\pi}^j(r'^j(r,g), g')$, denoted $\mathbb{E}\boldsymbol{\pi}^j$ for short. Repeat for r', giving $\mathbb{E}\boldsymbol{r}^j$.

3. Using the Phillips curve, calculate y:

$$y^j(r,g) = \frac{1}{\kappa}\left(\pi^j(r,g) - \mathbb{E}^j\boldsymbol{\pi}\right).$$

4. Construct one-step ahead output gap expectations, $\mathbb{E}\boldsymbol{y}^j$.

5. Construct the deposit rate function $r_d(r,g) = \max(0, r'^j(r,g))$.

6. Using the IS and Phillips curve, re-calculate y and π, respectively:

$$y^*(r,g) = \mathbb{E}\boldsymbol{y}^j - \sigma^{-1}\left(r_d(r,g) - \mathbb{E}\boldsymbol{\pi}^j - g\right) - \phi\left(r_d(r,g) - r'^j(r,g)\right),$$
$$\pi^*(r,g) = \beta\mathbb{E}\boldsymbol{\pi}^j + \kappa y^*(r,g),$$

and then update expectations, $\mathbb{E}\boldsymbol{y}^*$ and $\mathbb{E}\boldsymbol{\pi}^*$.

7. Construct numerical derivatives of π as follows:

$$\pi_1(r,g) \equiv \frac{\partial\pi^*(r,g)}{\partial r} = \begin{cases} \frac{\pi^*(i_k,g)-\pi^*(i_{k-1},g)}{i_k - i_{k-1}} & \text{for} \quad k = 1, ..., N_i, \\ \frac{\pi^*(i_1,g)-\pi^*(i_0,g)}{i_1 - i_0} & \text{for} \quad k = 0. \end{cases}$$

and denote the function π_1 for short. Calculate the one-step ahead values of these derivative functions, $\boldsymbol{\pi}_1(r'^j(r,g), g')$, and calculate expectations, denoted $\mathbb{E}\boldsymbol{\pi}_1$. Repeat for y giving $\mathbb{E}\boldsymbol{y}_1$.

8. Using the FOC equation to re-calculate r':

`for` $r'^{j}(r,g) > 0$,

$$r'^{*}(r,g) = \frac{1}{\psi(1+\beta)} \begin{pmatrix} \psi r + \psi\beta\mathbb{E}\boldsymbol{r}^{j} - (1-\psi)\beta\mathbb{E}\boldsymbol{\pi}_1\pi^{*}(r,g) + \zeta^{*}_{ZLB}(r,g) \\ -(1-\psi)\left(\mathbb{E}\boldsymbol{y}_1 + \sigma^{-1}\mathbb{E}\boldsymbol{\pi}_1 - \sigma^{-1}\right)\left(\lambda y^{*}(r,g) + \kappa\pi^{*}(r,g)\right) \end{pmatrix}$$

`else`

$$r'^{*}(r,g) = \frac{1}{\psi(1+\beta)} \begin{pmatrix} \psi r + \psi\beta\mathbb{E}\boldsymbol{r}^{j} - (1-\psi)\beta\mathbb{E}\boldsymbol{\pi}_1\pi^{*}(r,g) + \zeta^{*}_{ZLB}(r,g) \\ -(1-\psi)\left(\mathbb{E}\boldsymbol{y}_1 + \sigma^{-1}\mathbb{E}\boldsymbol{\pi}_1 + \phi\right)\left(\lambda y^{*}(r,g) + \kappa\pi^{*}(r,g)\right) \end{pmatrix}$$

9. `if` $\max\left(\left(\pi^{*}(r,g) - \pi^{j}(r,g)\right), \left(r' * (r,g) - r'^{j}(r,g)\right)\right) < \epsilon$, then `stop`.
`else` $j = j+1$ and update the guess as follows:

$$\pi^{j}(r,g) = \mathtt{u}\pi^{j-1}(r,g) + (1-\mathtt{u})\,\pi^{*}(r,g),$$
$$r'^{j}(r,g) = \mathtt{u}r'^{j-1}(r,g) + (1-\mathtt{u})\,r'^{*}(r,g).$$

Repeat steps 2-9.

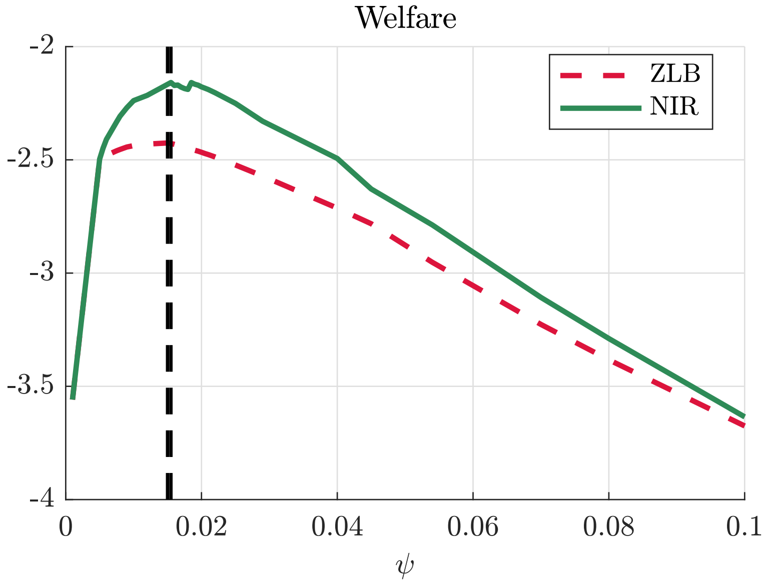
A.7 Numerical results: welfare computation [Section 2.4]

The social welfare function can be translated into a consumption equivalent measure via

$$CE = 100 \times (1-\beta)\,\lambda^{-1}\left(\sigma^{-1}+\eta\right)\mathbb{E}\left(V^{SW}\right), \tag{A25}$$

where η is the inverse labor supply elasticity, set to 0.47 in our calibration, and $\mathbb{E}\left(V^{SW}\right)$ is the unconditional mean of the social welfare function. CE is the percentage of steady state consumption that the representative household would forgo in each period to avoid uncertainty. Less negative values thus represent an improvement in welfare. Figure A4 plots the consumption equivalent measure of welfare across a range of values for the smoothing parameter, ψ. It demonstrates three features. One, allowing for negative interest rates in the toolkit of the policymaker is weakly welfare dominant. Two, it is optimal to delegate policy to a central banker with a small but meaningful preference for smoothing. Three, the optimal value of ψ is virtually the same, irrespective of whether negative interest rates are available or not.

Figure A4: Welfare and the optimal degree of smoothing

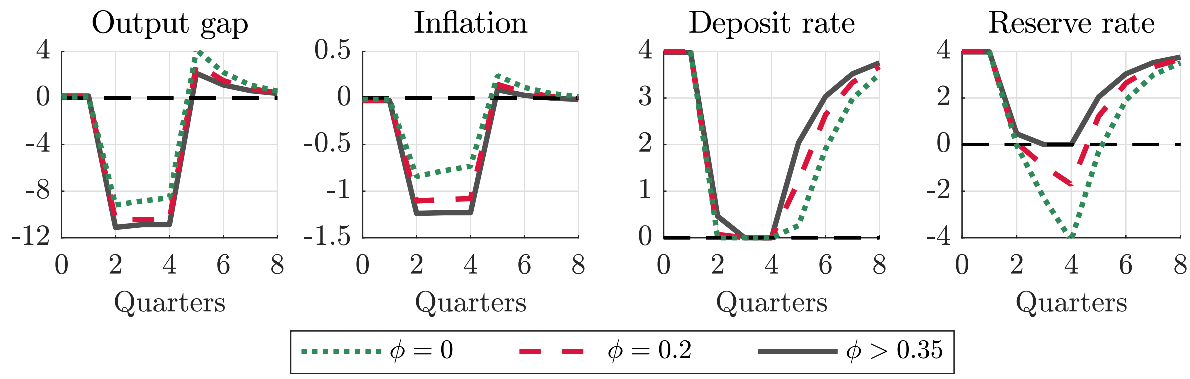


Note: Consumption equivalent in percent of steady state consumption. Black-dash is the optimal value of ψ. "ZLB" denotes policy without negative interest rates. "NIR" denotes policy with negative rates.

A.8 Numerical results: optimal policy experiment [Section 2.4]

In this section, we add one more set of results to our investigation regarding the optimality of negative rates. Figure A5 shows an experiment in which the natural real rate, s_t, drops into negative territory and remains at that level for 3 quarters before returning to steady state. The red-dash line is our baseline parameterization. The black-solid line is the equilibrium outcome when the policymaker is not able to set a negative reserve rate (or, equivalently, when the cost of negative interest rates is sufficiently high—in this case $\phi > 0.35$—such that the policymaker chooses not to use negative interest rates). The green-dotted line plots an extreme scenario where there is no cost of negative interest rates ($\phi = 0$).

Figure A5: Optimal policy scenarios



Note: Impulse responses to a drop in s_t into negative territory for 3 quarters before jumping back to its steady state value. The output gap is measured in percent. Inflation is in annualized percent deviation from steady state. The deposit and reserve rates are in levels, annualized.

When $\phi > 0.35$, the policymaker behaves as if there was a ZLB on the reserve rate. The nominal reserve rate is lowered to the ZLB, but this easing does not generate a sufficient fall in the real deposit rate, $r_{d,t} - \mathbb{E}_t\pi_{t+1}$, to offset the fall in s_t. As a result, inflation falls and the output gap opens. In contrast, when $\phi = 0.2$ the policymaker gradually lowers the reserve rate into negative territory, reaching -1.2% in period 4. Although the deposit rate remains bounded by zero, this negative reserve rate ensures that the deposit rate is lower after period 4 than without negative interest rates. This lower path for the deposit rate allows inflation to overshoot after s_t is back at steady state, also lowering the expected real deposit rate in early periods. As a consequence the drop in inflation and the widening of the output gap is less severe. The scenario without the cost of negative rates ($\phi = 0$) shows the maximum impact of negative interest rates. In this case, the reserve rate reaches -3.8% in period 2 and the deposit rate is a full 1 percentage point lower in period 6 than in the case without negative rates. The drop in the output gap and inflation is much less pronounced than in the other two scenarios.

This exercise illustrates that the increased frequency at the ZLB arises for two reasons: First, signalling with negative rates keeps the deposit rate lower-for-longer in response to a contractionary shock. Second, on impact the policymaker with access to negative rates is willing to cut the policy rate faster. Observe that, due to smoothing, the black-solid line does not reach the ZLB until period 3 as the benefit of cutting the period-2 policy rate further is outweighed by the cost in terms of smoothing rates. In contrast, the red-dash and green-dot lines (negative rate scenarios) already reach the ZLB in period 2.

A.9 Comparative statics: closed-form solutions [Section 2.4]

In Section 2.4 we set $\lambda = 0$, we set $\psi = 0$ except for between periods 1 and 2, and $s_t = 0$ for $t > 1$. This allows for an analytical derivation of equilibrium outcomes. In particular, $\pi_t = y_t = 0$ for $t > 2$. Thus, the central banks loss function reduces to

$$-V \propto \pi_1^2 + \beta\left((1-\psi)\,\pi_2^2 + \psi\,(r_2 - r_1)^2\right). \tag{A26}$$

The policymaker is subject to the following constraints

$$\pi_1 = \beta\pi_2 + \kappa y_1, \tag{A27}$$

$$y_1 = y_2 - \sigma^{-1}\left(r_{d,1} - \pi_2\right) - \phi\left(r_{d,1} - r_1\right) + g, \tag{A28}$$

$$\pi_2 = \kappa y_2, \tag{A29}$$

$$y_2 = -\sigma^{-1} r_2, \tag{A30}$$

$$r_{d,1} \geq -\bar{r}, \tag{A31}$$

$$r_2 \geq -\bar{r}, \tag{A32}$$

$$r_{d,1} - r_1 \geq 0, \tag{A33}$$

$$\left(r_{d,1} + \bar{r}\right)\left(r_{d,1} - r_1\right) = 0, \tag{A34}$$

where the expectations operator has been dropped because there is no uncertainty. In addition, there is no incentive to set a negative interest rate in period 2 so $r_{d,2} = r_2$. In contrast to the main text, we make g mean zero and set the ZLB constraint as $-\bar{r}$.

We consider optimal policy under discretion. There are 4 possible equilibrium outcomes:

$$(++): \quad r_1 > -\bar{r}, \quad r_2 > -\bar{r}, \tag{A35}$$

$$(0+): \quad r_1 = -\bar{r}, \quad r_2 > -\bar{r}, \tag{A36}$$

$$(-+): \quad r_1 < -\bar{r}, \quad r_2 > -\bar{r}, \tag{A37}$$

$$(-0): \quad r_1 < -\bar{r}, \quad r_2 = -\bar{r}. \tag{A38}$$

We solve the problem backwards. First solving for the optimal r_2 given a value for r_1.

For $(\cdot 0)$, we have

$$r_2^{*(0)} = -\bar{r}. \tag{A39}$$

For $(\cdot +)$, the period 2 problem is given by

$$\min_{r_2} \quad (1-\psi)\,\pi_2^2 + \psi\,(r_2 - r_1)^2 \quad \text{s.t.} \quad \pi_2 = -\kappa\sigma^{-1} r_2. \tag{A40}$$

The first-order condition is given by

$$(1-\psi)\left(\kappa\sigma^{-1}\right)^2 r_2 + \psi\,(r_2 - r_1) = 0, \tag{A41}$$

or, rearranged, as

$$r_2^{*(+)} = R_2^{(+)} r_1, \tag{A42}$$

$$\pi_2^{*(+)} = \Pi_2^{(+)} r_1, \tag{A43}$$

$$\text{where} \quad R_2^{(+)} \equiv \frac{\psi}{\psi + (1-\psi)\left(\kappa\sigma^{-1}\right)^2}, \tag{A44}$$

$$\Pi_2^{(+)} \equiv -\kappa\sigma^{-1} R_2^{(+)}. \tag{A45}$$

Now that we have the optimal reaction function for r_2 as a function of r_1, we can solve the period 1 problem, taking the behaviour of the policymaker in period 2 as given.

For (++), the period 1 problem is given by

$$\min_{r_1} \quad \pi_1^2 + \beta\left((1-\psi)\,\pi_2^2 + \psi\,(r_2 - r_1)^2\right) \tag{A46}$$

$$\text{s.t.} \quad \pi_1 = \Pi_1^{(++)} r_1 + \kappa g, \tag{A47}$$

$$\pi_2 = \Pi_2^{(+)} r_1, \tag{A48}$$

$$r_2 = R_2^{(+)} r_1, \tag{A49}$$

$$\text{where} \quad \Pi_1^{(++)} \equiv -\kappa\left(\left(\beta + 1 + \kappa\sigma^{-1}\right)\sigma^{-1} R_2^{(+)} + \sigma^{-1}\right), \tag{A50}$$

and the first-order condition is given by

$$\left(\Pi_1^{(++)} r_1 + \kappa g\right)\Pi_1^{(++)} + \beta\left((1-\psi)\left(\Pi_2^{(+)}\right)^2 r_1 + \psi\left(R_2^{(+)} - 1\right)^2 r_1\right) = 0, \tag{A51}$$

or, rearranged, as

$$r_1^{*(++)} = -\frac{\kappa\Pi_1^{(++)} g}{\left(\Pi_1^{(++)}\right)^2 + \beta\left((1-\psi)\left(\Pi_2^{(+)}\right)^2 + \psi\left(R_2^{(+)} - 1\right)^2\right)}. \tag{A52}$$

For $(-+)$, the constraints are given by

$$\pi_1 = \Pi_1^{(-+)} r_1 + C\Pi_1^{(-+)} \bar{r} + \kappa g, \tag{A53}$$
$$\pi_2 = \Pi_2^{(+)} r_1, \tag{A54}$$
$$r_2 = R_2^{(+)} r_1, \tag{A55}$$
$$\text{where} \quad \Pi_1^{(-+)} \equiv -\kappa \left(\left(\beta + 1 + \kappa\sigma^{-1} \right) \sigma^{-1} R_2^{(+)} - \phi \right), \tag{A56}$$
$$C\Pi_1^{(-+)} \equiv \kappa \left(\sigma^{-1} + \phi \right), \tag{A57}$$

and the solution is given by

$$r_1^{*(-+)} = -\frac{C\Pi_1^{(-+)}\Pi_1^{(-+)}\bar{r} + \kappa\Pi_1^{(-+)} g}{\left(\Pi_1^{(-+)}\right)^2 + \beta\left((1-\psi)\left(\Pi_2^{(+)}\right)^2 + \psi\left(R_2^{(+)} - 1\right)^2\right)}. \tag{A58}$$

For $(0+)$, we have

$$r_1^{*(0+)} = -\bar{r}. \tag{A59}$$

For (-0), the constraints are given by

$$\pi_1 = \Pi_1^{(-0)} r_1 + C\Pi_1^{(-0)} \bar{r} + \kappa g, \tag{A60}$$
$$\pi_2 = C\Pi_2^{(0)} \bar{r}, \tag{A61}$$
$$r_2 = -\bar{r}, \tag{A62}$$
$$\text{where} \quad \Pi_1^{(-0)} \equiv \kappa\phi, \tag{A63}$$
$$C\Pi_1^{(-0)} \equiv \kappa \left(\left(\beta + 2 + \kappa\sigma^{-1} \right) \sigma^{-1} + \phi \right), \tag{A64}$$
$$C\Pi_2^{(0)} \equiv \kappa\sigma^{-1}, \tag{A65}$$

and the first-order condition is given by

$$\left(\Pi_1^{(-0)} r_1 + C\Pi_1^{(-0)}\bar{r} + \kappa g\right) \Pi_1^{(-0)} + \beta\psi\left(\bar{r} + r_1\right) = 0, \tag{A66}$$

or, rearranged, as

$$r^{*(-0)} = -\frac{\Pi_1^{(-0)} C\Pi_1^{(-0)}\bar{r} + \Pi_1^{(-0)}\kappa g + \beta\psi\bar{r}}{\left(\Pi_1^{(-0)}\right)^2 + \beta\psi}. \tag{A67}$$

This completes the full set of equilibrium conditions. Numerically, we solve for each possible case and throw out any solutions which violate the assumptions of that case. If multiple solutions exist, we choose the one that maximizes welfare.

Next, we use these analytical results to prove Propositions 5 and 6 in the main text.

Proof of Proposition 5 Proposition 5 states that there exists a threshold ϕ^* below which a negative interest rate policy is effective in both raising inflation π_1 and output y_1. We prove this as follows.

First, we assume the size of g ensures that $r_2 = r_{d,2} > 0$, $r_{d,1} = 0$, and $r < 0$. Second, r_2 is set optimally as in equation (A42). Third, we substitute into the period 1 Phillips curve in order to write π_1 in terms of r_1. This is given by

$$\pi_1 = -\kappa \left(\left(\beta + 1 + \kappa\sigma^{-1}\right) \sigma^{-1} \frac{\psi}{\psi + (1-\psi)\left(\kappa\sigma^{-1}\right)^2} - \phi \right) r_1. \tag{A68}$$

The condition for negative rates to be effective, $\partial\pi_1/\partial r_1 < 0$, therefore holds when

$$\phi < \phi_\pi^* = \left(\beta + 1 + \kappa\sigma^{-1}\right) \sigma^{-1} \frac{\psi}{\psi + (1-\psi)\left(\kappa\sigma^{-1}\right)^2}. \tag{A69}$$

Note the threshold for raising output in period 1, ϕ_y^*, is more demanding. In particular,

$$\phi_y^* = \left(1 + \kappa\sigma^{-1}\right) \sigma^{-1} \frac{\psi}{\psi + (1-\psi)\left(\kappa\sigma^{-1}\right)^2}. \tag{A70}$$

Hence, it is possible, if $\phi_y^* < \phi < \phi_\pi^*$, that negative rates raise inflation while causing a contraction in output. Setting $\phi^* = \phi_y^*$ completes the proof. ■

Proof of Proposition 6 Proposition 6 states that the cut in the policy rate needed to generate the same effect on output (and inflation) is larger in negative than in positive territory. We prove this as follows.

One, the IS equation in period 1 can be rewritten as a relationship between y_1 and g,

$$y_1(g) = y_2(r_1(g)) - \sigma^{-1}(r_{d,1}(r_1(g)) - \pi_2(r_1(g))) - \phi(r_{d,1}(r_1(g)) - r_1(g)) + g. \quad \text{(A71)}$$

Two, note that $\partial r_{d,1}/\partial r_1 = 1$ when $r_1 > 0$ and $\partial r_{d,1}/\partial r_1 = 0$ when $r_1 < 0$. Three, since we assume g is such that $r_2 > 0$, it follows that $\partial \pi_2/\partial r_1$ and $\partial y_2/\partial r_1$ are common across both $r_1 < 0$ and $r_1 > 0$ scenarios. Four, note that $\partial \pi_1/\partial r_1$ only differs across scenarios in so far as $\partial y_1/\partial r_1$ differs across scenarios. Hence, when evaluating the effectiveness of policy, we need only concern ourselves with $\partial y_1/\partial r_1$. Five, let us evaluate the response $\partial r_1/\partial g$ that ensures $dy_1/dg = 0$. When $r_1 < 0$ and $r_2 > 0$, the derivative $\frac{dy_1}{dg} = 0$ is

$$0 = \frac{\partial y_2}{\partial r_1}\frac{\partial r_1}{\partial g} - \sigma^{-1}\left(-\frac{\partial \pi_2}{\partial r_1}\frac{\partial r_1}{\partial g}\right) - \phi\left(-\frac{\partial r_1}{\partial g}\right) + 1, \quad \text{(A72)}$$

$$\implies \left.\frac{\partial r_1}{\partial g}\right|_{r_1<0, r_2>0} = \frac{1}{\left(-\frac{\partial y_2}{\partial r_1} - \sigma^{-1}\frac{\partial \pi_2}{\partial r_1} - \phi\right)}. \quad \text{(A73)}$$

In the scenario where $r_1, r_2 > 0$, the derivative is given by

$$0 = \frac{\partial y_2}{\partial r_1}\frac{\partial r_1}{\partial g} - \sigma^{-1}\left(\frac{\partial r_1}{\partial g} - \frac{\partial \pi_2}{\partial r_1}\frac{\partial r_1}{\partial g}\right) + 1, \quad \text{(A74)}$$

$$\implies \left.\frac{\partial r_1}{\partial g}\right|_{r_1, r_2>0} = \frac{1}{\left(-\frac{\partial y_2}{\partial r_1} - \sigma^{-1}\frac{\partial \pi_2}{\partial r_1} + \sigma^{-1}\right)}. \quad \text{(A75)}$$

Next, we assume that $-\left(\frac{\partial y_2}{\partial r_1} + \sigma^{-1}\frac{\partial \pi_2}{\partial r_1}\right) > \phi$. This is equivalent to the threshold condition in Proposition 5 that ensures negative rates are effective. If this condition holds and negative rates are effective, then the proof reduces to $\sigma^{-1} > -\phi$, which is always true.

Finally, note that the proof follows the same steps if started from $d\pi_1/dg = 0$. ■

B Quantitative model

Appendix B relates to Section 3 on the effectiveness of negative rates in a quantitative new-Keynesian model. Section B.1 shows how to derive the financial sector equilibrium in just two equations. Section B.2 documents the complete set of equilibrium equations. Section B.3 provides further information on the parameterization of the model regarding calibration targets, estimation method and results, as well as data sources and treatment. Section B.4 reports additional empirical evidence on policy smoothing. Section B.5 shows that without interest rate inertia the signalling channel is not active. Section B.6 derives the bank profit decomposition in our baseline model and in an extended version of the model with firm equity and loan finance. Sections B.7 documents the robustness of our results with respect to changes in the Frisch labor supply elasticity, the Phillips curve slope, the investment elasticity, and the introduction of nominal wage rigidities. Finally, Section B.8 summarizes the necessary changes to the equilibrium system of equations when nominal wage rigidities are introduced.

B.1 Set up: derivation of the banker's problem [Section 3.1]

A banker j solves

$$V_{n,t}(j) = \max_{\{S_t(j),A_t(j),D_t(j),N_t(j)\}} \mathbb{E}_t \Lambda_{t,t+1} \left((1-\theta) N_{t+1}(j) + \theta V_{n,t+1}(j) \right), \tag{B1}$$

subject to

$$Q_t S_t(j) + A_t(j) = D_t(j) + N_t(j), \tag{B2}$$

$$V_{n,t}(j) \geq \lambda Q_t S_t(j), \tag{B3}$$

$$A_t(j) = \alpha(x_t) D_t(j), \tag{B4}$$

$$N_t(j) = R_{k,t} Q_{t-1} S_{t-1}(j) + \frac{R_{t-1}}{\Pi_t} A_{t-1}(j) - \frac{R_{d,t-1}}{\Pi_t} D_{t-1}(j),, \tag{B5}$$

where the constraints are the balance sheet constraint, incentive compatibility constraint, reserve ratio, and net worth accumulation, respectively. We calibrate the model such that the incentive constraint is always binding. Next, we simplify the system of constraints by substituting reserves, $A_t(j)$, and deposits, $D_t(j)$, making use of Equations (B2) and (B4). We also define $\Phi_t \equiv Q_t S_t(j)/N_t(j)$ to be the leverage ratio of a banker (and Φ_t is common across banks). Thus, the accumulation of net worth, (B5), is given by

$$N_t(j) = \left(R_{k,t} \Phi_{t-1} - \frac{R_{d,t-1} - \alpha(x_t) R_{t-1}}{(1-\alpha(x_t)) \Pi_t} (\Phi_{t-1} - 1) \right) N_{t-1}(j). \tag{B6}$$

Furthermore, we conjecture the value function to take the form

$$V_{n,t}(j) = (\zeta_{s,t} \Phi_t + \zeta_{n,t}) N_t(j), \tag{B7}$$

where $\zeta_{s,t}$ and $\zeta_{n,t}$ are as yet undetermined.

Substituting (B6) and (B7), the banker's problem can be rewritten as

$$\begin{aligned}(\zeta_{s,t} \Phi_t + \zeta_{n,t}) = \max_{\Phi_t} \mathbb{E}_t \Lambda_{t,t+1} & \left((1-\theta) + \theta (\zeta_{s,t+1} \Phi_{t+1} + \zeta_{n,t+1}) \right) \\ & \times \left(R_{k,t+1} \Phi_t - \frac{R_{d,t} - \alpha(x_{t+1}) R_t}{(1-\alpha(x_{t+1})) \Pi_{t+1}} (\Phi_t - 1) \right), \end{aligned} \tag{B8}$$

subject to

$$\zeta_{s,t} \Phi_t + \zeta_{n,t} = \lambda \Phi_t. \tag{B9}$$

We rearrange the incentive compatibility constraint (B9) and iterate one period forward to find optimal (and maximum) leverage given by

$$\Phi_{t+1} = \frac{\zeta_{n,t+1}}{\lambda - \zeta_{s,t+1}}. \tag{B10}$$

With (B10), comparing the left and right hand side of (B8), we verify the conjectured functional form of the value function. This allows us to summarize the solution to the financial intermediary's problem in the binding incentive constraint given by

$$\lambda\Phi_t = \mathbb{E}_t\Lambda_{t,t+1}\left((1-\theta)+\theta\lambda\Phi_{t+1}\right)\left(R_{k,t+1}\Phi_t - \frac{R_{d,t}-\alpha\left(x_{t+1}\right)R_t}{\left(1-\alpha\left(x_{t+1}\right)\right)\Pi_{t+1}}\left(\Phi_t-1\right)\right). \tag{B11}$$

Aggregate net worth in the financial sector evolves as a weighted sum of existing banks' accumulated net worth (B6) and start up funds new banks receive from the household. Entering banks receive a fraction ω of the total value of intermediated assets, i.e. $\omega Q_t S_{t-1}$. In equilibrium, $S_t = K_t$. Thus, the evolution of aggregate net worth is given by

$$N_t = \theta\left(R_{k,t}\Phi_{t-1} - \frac{R_{d,t-1}-\alpha\left(x_t\right)R_{t-1}}{\left(1-\alpha\left(x_t\right)\right)\Pi_t}\left(\Phi_{t-1}-1\right)\right)N_{t-1} + \omega Q_t K_{t-1}. \tag{B12}$$

Equations (B11) and (B12) express the financial sector problem in just two equations. This completes the derivation.

B.2 Set up: list of equilibrium conditions [Section 3.1]

In equilibrium, we summarize the quantitative model in 23 equations in 23 endogenous variables, $\{Y_t,\, Y_{m,t},\, L_t,\, C_t,\, \tilde{C}_t,\, \Lambda_{t,t+1},\, \mu_t,\, K_t,\, I_t,\, I_{n,t},\, N_t,\, \Phi_t,\, \Delta_t,\, W_t,\, \Pi_t,\, X_t,\, P_{m,t},\, Q_t,\, R_{k,t},\, R_{T,t},\, R_t,\, R_{d,t},\, CS_t\}$, and 3 exogenous processes, $\{\zeta_t,\, \epsilon_t,\, \varepsilon_{m,t}\}$. Government expenditure, G, is financed via lump-sum taxes and kept constant.

Households

- Euler equation

$$1 = \mathbb{E}_t \Lambda_{t,t+1} \exp\left(\zeta_t\right) R_{d,t} / \Pi_{t+1} \tag{B13}$$

- Labor supply

$$\mu_t W_t = \chi L_t^{\varphi} \tag{B14}$$

- Stochastic discount factor

$$\Lambda_{t,t+1} = \beta \mu_{t+1} / \mu_t \tag{B15}$$

- Marginal utility of consumption

$$\mu_t = \tilde{C}_t^{-\sigma} - \beta \hbar \mathbb{E}_t \tilde{C}_{t+1}^{-\sigma} \tag{B16}$$

Financial intermediaries

- Incentive compatibility constraint

$$\lambda \Phi_t = \mathbb{E}_t \Lambda_{t,t+1} \left(\left(1-\theta\right) + \theta \lambda \Phi_{t+1}\right) \left(R_{k,t+1} \Phi_t - \frac{R_{d,t} - \alpha\left(x_t\right) R_t}{\left(1-\alpha\left(x_t\right)\right) \pi_{t+1}} \left(\phi_t - 1\right) \right) \tag{B17}$$

- Evolution of aggregate net worth

$$N_t = \theta \left(R_{k,t} \Phi_{t-1} - \frac{R_{d,t-1} - \alpha\left(x_t\right) R_{t-1}}{\left(1-\alpha\left(x_t\right)\right) \Pi_t} \left(\Phi_{t-1} - 1\right) \right) N_{t-1} + \omega Q_t K_{t-1} \tag{B18}$$

Intermediate goods firms

- Price of capital

$$\begin{aligned} 1 = Q_t & \left(1 - \frac{\eta}{2} \left(\frac{I_{n,t} - I_{n,t-1}}{I_{n,t-1} + I} \right)^2 - \eta \frac{I_{n,t} - I_{n,t-1}}{\left(I_{n,t-1} + I\right)^2} I_{n,t} \right) \\ & + \mathbb{E}_t \Lambda_{t,t+1} Q_{t+1} \left(\eta \left(I_{n,t+1} - I_{n,t}\right) \frac{I_{n,t+1} + I}{\left(I_{n,t} + I\right)^3} I_{n,t+1} \right) \end{aligned} \tag{B19}$$

- Production function

$$Y_{m,t} = K_{t-1}^{\gamma} L_t^{1-\gamma} \tag{B20}$$

- Labor demand

$$W_t = (1-\gamma)\, P_{m,t} Y_{m,t} / L_t \tag{B21}$$

- Return on capital

$$R_{k,t} = \frac{\gamma P_{m,t} Y_{m,t} / K_{t-1} + Q_t - \delta}{Q_{t-1}} \tag{B22}$$

Retail firms

- Price Phillips curve

$$\left(\frac{\epsilon}{\epsilon-1}\right) \frac{\mathcal{D}_t}{\mathcal{F}_t} = \left(\frac{1-\iota\,\Pi_t^{\epsilon-1}}{1-\iota}\right)^{\frac{1}{1-\epsilon}}. \tag{B23}$$

$$\text{where} \quad \mathcal{D}_t \equiv \mu_t P_{m,t} Y_t + \beta\iota \mathbb{E}_t\, \Pi_{t+1}^{\epsilon} \mathcal{D}_{t+1}, \qquad \mathcal{F}_t \equiv \mu_t Y_t + \beta\iota \mathbb{E}_t\, \Pi_{t+1}^{\epsilon-1} \mathcal{F}_{t+1}.$$

- Price dispersion

$$\Delta_t = (1-\iota) \left[\left(\frac{\epsilon}{\epsilon-1}\right) \frac{\mathcal{D}_t}{\mathcal{F}_t}\right]^{-\epsilon} + \iota \Pi_t^{\epsilon} \Delta_{t-1} \tag{B24}$$

Monetary policy

- Policy rule

$$R_{T,t} = \left(R \Pi_t^{\phi_\pi} \left(\frac{X_t}{X}\right)^{\phi_x}\right)^{1-\rho} R_{t-1}^{\rho} \exp\left(\varepsilon_{m,t}\right) \tag{B25}$$

- No arbitrage

$$\begin{aligned} &\text{(I)} \quad R_t = R_{d,t} = R_{T,t}\ , \text{ or} \\ &\text{(II)} \quad R_t = R_{d,t} = \max\left\{1, R_{T,t}\right\}, \text{ or} \\ &\text{(III)} \quad R_t = R_{T,t} \quad \text{and} \quad R_{d,t} = \max\left\{1, R_{T,t}\right\}. \end{aligned} \tag{B26}$$

General equilibrium

- Aggregate output

$$Y_t = Y_{m,t} / \Delta_{p,t}, \tag{B28}$$

- Aggregate resource constraint

$$Y_t = C_t + I_t + G \tag{B29}$$

- Capital accumulation

$$K_t = K_{t-1} + f\left(I_{n,t}, I_{n,t-1}\right), \tag{B30}$$

$$\text{where} \quad f\left(I_{n,t}, I_{n.t-1}\right) \equiv \left(1 - (\eta/2)\left(\left(I_{n,t} + I_{n,t-1}\right) / \left(I_{n,t-1} + I\right)\right)^2\right) I_{n,t}.$$

Further definitions

- Habit adjusted consumption

$$\tilde{C}_t = C_t - \hbar C_{t-1} \tag{B31}$$

- Total investment

$$I_t = I_{n,t} + \delta K_{t-1} \tag{B32}$$

- Leverage

$$\Phi_t = Q_t K_t / N_t \tag{B33}$$

- Marginal cost

$$X_t = P_{m,t} \tag{B34}$$

- Credit spread

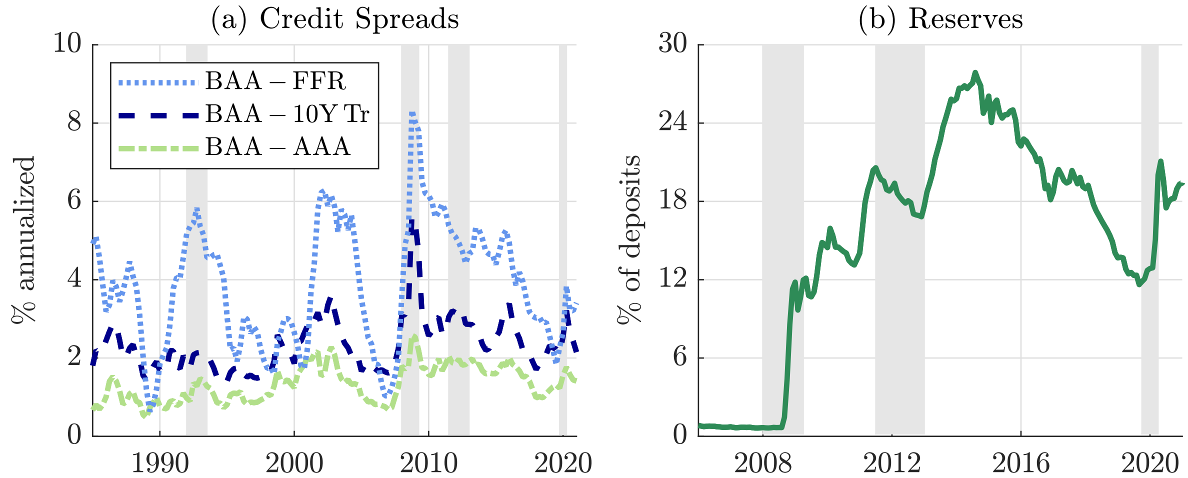
$$CS_t = R_{k,t+1} / \left(R_{d,t} / \Pi_{t+1}\right) \tag{B35}$$

B.3 Parameterization: further details [Section 3.2]

Table 1 in the main text presents the baseline parameterization of the quantitative model. The parameters are grouped into three blocks. Block A contains structural parameters that are assigned standard values from the literature. Block B is calibrated using steady state relationships. Block C is estimated using a simulated method of moments procedure.

Block B. *Leverage* A good data counterpart to aggregate leverage in the model is hard to come by. From 2009 to 2019, the US commercial banking sector had an average leverage of 9.4.[28] This measure excludes non-bank financial institutions such as hedge funds and broker dealers that are typically more leveraged. In 2021, estimates for the total assets of the non-bank financial sector were 1.86 times larger than the total assets of commercial banks. Moreover, from 2009 to 2019, leverage of the non-financial corporate business sector was 1.9, implying a significantly lower economy-wide leverage ratio. We follow Gertler and Karadi (2011), aggregating across these highly heterogeneous sectors and, assuming that leverage in the non-bank financial sector is twice that of the commercial banking sector, end up with a conservative estimate of aggregate leverage of 3.6. Given the uncertainty in these calculations, we opt to calibrate the model to a leverage ratio of 4 (see below for further details on how we construct our measure of leverage).

Figure B1: Credit spreads and reserves in the US



Note: (a) AAA and BAA are Moody's Seasoned AAA and BAA Corporate Bond Yields, respectively; FFR is the Effective Federal Funds Rate; 10Y Tr is the market yield on Treasury Securities at 10-Year Constant Maturity. (b) Total reserves of depository institutions over total deposits of commercial banks. Source: Federal Reserve Bank of St Louis.

[28] Consistent with the model, leverage is $A/(A-L)$, where A is total assets and L is total liabilities.

Block B. *Credit spreads* Calibrating the steady state credit spread is equally tricky. In Figure B1(a) we plot three alternative spread measures used in the literature. The first is the spread between the BAA corporate bond yield and the federal funds rate (light blue-dot). The two component interest rates that compromise the spread are a reasonable match for the expected return on capital and the short-term policy rate in the model, respectively. We thus use the cyclical properties of these series in the estimation stage below. However, for matching the steady state credit spread, this measure is not ideal because it contains a maturity mismatch. The corporate bonds yields are based on long-term bonds with a maturity of 20 years and above whereas the federal funds rate is a short-term rate. Thus, this series is likely to contain both a liquidity and term premium in addition to a pure risk premium. To get a sense of these various premia, we plot the spread between the BAA corporate bond yield and the 10 year Treasury yield (dark blue-dash) and between the BAA and AAA corporate bond yields (green dot-dash), respectively. For the credit spread in the model, we match its steady state to 1% annualized which corresponds to the mean of the "BAA-AAA" series over the sample period. This series is generally perceived to be a good empirical measure of the safety or quality premium that we capture with the financial friction in our model (see Krishnamurthy and Vissing-Jorgensen, 2012).

Block B. *Reserve ratio* We set the reserve-to-deposit rate $\alpha = 0.2$. This value is broadly in line with data for both the euro area—as displayed in Figure 1—and the United States. Figure B1(b) shows the evolution of the US reserve ratio. In the aftermath of the 2007/08 financial crisis, total reserve holdings strongly increased, reflecting banks' desire to hedge against heightened liquidity risk and the Federal Reserve's willingness to supply extensive additional reserves to the banking system via a range of liquidity and QE programs. Accordingly, the reserve-to-deposit ratio rose from a pre-crisis level of around 1% to a peak of 27.9% in August 2014. The banking system's demand for liquidity spiked again during the Covid-19 crisis when the Federal Reserve once more sharply increased the provision of reserves to meet this additional demand. Overall, we find a value of 18.9% for the average reserve ratio over the post-financial crisis period in the US. As the strength of the costly interest margin channel of negative interest rates will depend sensitively on the quantity of reserves in the banking system, in Section 3.4 in the main text we conduct a sensitivity analysis where we vary this quantity and show the implications on the effectiveness of a negative interest rate policy.

Block C. We estimate the structural parameters in Block C following the method of simulated moments in Basu and Bundick (2017). In particular, the parameter values are chosen to minimize the distance between the model implied moments and their data counterparts. Formally, the vector of estimated parameters, Θ, is the solution to

$$\min_{\Theta} \left(\mathcal{H}^D - \mathcal{H}\left(\Theta\right)\right)' \mathcal{W}^{-1} \left(\mathcal{H}^D - \mathcal{H}\left(\Theta\right)\right), \tag{B36}$$

where $\mathcal{H}^D$ is a vector of data moments, $\mathcal{H}\left(\Theta\right)$ denotes its model counterpart, and $\mathcal{W}$ is a diagonal weighting matrix containing the standard errors of the estimated data moments.

The estimation targets ten moments from US time-series data and five yield curve moments. The first ten moments are the standard deviations and autocorrelations of output, consumption, inflation, the federal funds rate, and the credit spread, respectively. The remaining five moments are the movements in the 6-month, 1-, 2-, 5-, and 10-year risk-free rates, respectively, relative to the movement in the 3-month risk-free rate in response to a monetary shock. Empirical estimates are taken from Altavilla et al. (2019). The risk-free yield curve can be extracted from the model using the following set of equations:

$$\begin{aligned} P_{2,t} &= \mathbb{E}_t \Lambda_{t,t+1} P_{1,t+1}, \\ &\vdots \\ P_{40,t} &= \mathbb{E}_t \Lambda_{t,t+1} P_{39,t+1}, \end{aligned}$$

where $P_{1,t} = 1/R_t$ is the price of a 1-period risk-free bond that pays 1 unit in period $t+1$. The annualized yield on the 10-year risk-free bond is therefore given by $R_{40,t} = P_{40,t}^{-1/10}$.

With 15 moments, we estimate four parameters $\theta = \{\eta, \rho, \sigma_\zeta, \sigma_\epsilon\}$, the inverse investment elasticity, the policy rule inertia coefficient, and the standard deviations of risk premium and cost-push innovations. The estimation is thus over-identified. We choose to estimate the investment elasticity parameter because its value is not well-informed by the literature and its value has implications for the strength of the financial accelerator and the dynamics of credit spreads and net worth. The estimation delivers an inverse investment elasticity of $\eta = 1.617$. We also choose to estimate the policy rule inertia coefficient because it is crucial for the strength of the signalling channel of negative interest rates. The estimation delivers a value of $\rho = 0.856$, which suggests a significant amount of policy smoothing.

Table B1 compares the parameterized model implied moments with those from the data. The table also includes the 95% confidence interval around the data estimates. Despite only estimating a small number of parameters, the model does a good job of matching the data. The model implied moments are within the confidence interval for the yield curve moments. In terms of the business cycle moments, the model does well in terms of matching most of the standard deviations but generates too much persistence relative to the data (the exception is the credit spread, in which the data is more persistent).

Table B1: Simulated method of moments results

	Data	Model		Data	Model		Data	Model
std(y)	1.014 (0.76-1.27)	0.877	ac(y)	0.874 (0.82-0.93)	0.973	mp(r_{6m})	0.843 (0.80-0.89)	0.839
std(c)	0.714 (0.54-0.89)	0.641	ac(c)	0.831 (0.77-0.89)	0.990	mp(r_{1y})	0.677 (0.55-0.81)	0.587
std(π)	0.175 (0.14-0.21)	0.196	ac(π)	0.330 (0.14-0.52)	0.760	mp(r_{2y})	0.503 (0.29-0.72)	0.301
std(r)	0.265 (0.20-0.33)	0.144	ac(r)	0.935 (0.89-0.98)	0.961	mp(r_{5y})	0.324 (0.11-0.54)	0.135
std(cs)	0.279 (0.20-0.36)	0.345	ac(cs)	0.895 (0.83-0.95)	0.745	mp(r_{10y})	0.092 (-0.08-0.26)	0.101
Untargeted moments								
std(i)	4.470 (2.92-6.02)	4.272	ac(i)	0.914 (0.84-0.99)	0.972	cr(y, c)	0.807 (0.72-0.89)	0.599
cr(y, i)	0.906 (0.86-0.95)	0.890	cr(y, π)	0.362 (0.14-0.58)	-0.539	cr(y, r)	0.689 (0.56-0.82)	-0.644
cr(y, cs)	-0.690 (-0.84–0.54)	-0.539						

Note: Construction of moments given in Appendix B.3. y, c, π, r, and cs refer to GDP, consumption, inflation, the federal funds rate, and the credit spread, respectively. std(·) and ac(·) refer to the standard deviation and first-order autocorrelation. $r_{6m}, r_{1y}, r_{2y}, r_{5y}$, and r_{10y} refers to the OIS 6 month, 1, 2, 5, and 10 year rate, respectively. mp(·) refers to the relative response of the relevant OIS rate to the 3 month OIS rate in response to a monetary policy shock. Estimates are taken from Altavilla et al. (2019).

Data sources. We use US quarterly data covering the period 1985:Q1 to 2019:Q1. All macroeconomic and financial time series used are extracted from the Federal Reserve Economic Data (FRED) database at the St Louis FED. Table B2 summarizes this.

Data treatment We transform all nominal aggregate quantities into real per-capita terms. Inflation is defined as the quarter-on-quarter log growth rate of the GDP deflator. Nominal interest rates and spreads are divided by four to generate quarterly rates. For the estimation, all variables are stationarized using a standard HP-filter ($\lambda = 1600$). Data moments are matched with model moments for all relevant observables, where a lower case denotes the log deviation of the corresponding variable from steady state. Table B3 documents the data transformations in detail.

Table B2: Data sources

Mnemonic	Description
CNP16OV	Population level
GDP	Gross domestic product
GDPDEF	Gross domestic product: implicit price deflator
GPDI	Gross private domestic investment
PCDG	Personal consumption expenditures: durable goods
PCND	Personal consumption expenditures: nondurable goods
PCESV	Personal consumption expenditures: services
FEDFUNDS	Effective federal funds rate
DGS10	10-Year Treasury constant maturity rate
AAA	Moody's seasoned Aaa corporate bond yield
BAA	Moody's seasoned Baa corporate bond yield
TOTRESNS	Total reserves of depository institutions
DPSACBM027NBOG	Deposits, all commercial banks
TABSNNCB	Total assets, nonfinancial corporate business
TLBSNNCB	Total liabilities, nonfinancial corporate business
TLAACBW027SBOG	Total assets, all commercial banks
TLBACBW027SBOG	Total liabilities, all commercial banks

Table B3: Data treatment

Observable	Description	Construction
Steady state calibration & Figure B1		
	Spread measure I	BAA - FEDFUNDS
	Spread measure II	BAA - DGS10
	Spread measure III	BAA - AAA
	Reserve ratio	TOTRESNS/DPSACBM027NBOG
	Leverage	*see computation below**
Dynamic moment matching		
y	Output	HP-filter[GDP/(GDPDEF x NCP160V)]
c	Consumption	HP-filter[(PCND + PCESV)/(GDPDEF x NCP160V)]
π	Inflation	HP-filter[ln(GDPDEF/GDPDEF$_{-1}$)]
r	Reserve rate	HP-filter[FEDFUNDS/4]
cs	Credit spread	HP-filter[(BAA - FEDFUNDS)/4]
i	Investment	HP-filter[(PCDG + GPDI)/(GDPDEF x NCP160V)]

* Construction of the leverage series:

$$\text{Aggregate Leverage}_t = \frac{A_t^{\text{cb}}(1+s) + A_t^{\text{nfc}}}{A_t^{\text{cb}}(1+s) + A_t^{\text{nfc}} - L_t^{\text{cb}} - L_t^{\text{ncbfi}} - L_t^{\text{nfc}}}, \tag{B37}$$

where A_t and L_t denote assets and liabilities and where the superscripts "cb", "nfc", and "ncbfi" refer to commercial banks, non-financial corporations, and non-commercial bank financial institutions, respectively. L_t^{ncbfi} is given by

$$L_t^{\text{ncbfi}} = sA_t^{\text{cb}}\left(1 - \frac{1}{f\left(\frac{A_t^{\text{cb}}}{A_t^{\text{cb}} - L_t^{\text{cb}}}\right)}\right) \tag{B38}$$

where $s = 1.86$ and we assume $f = 2$.[29]

[29] The scaling factor s is derived from the May 2021 Federal Reserve Financial Stability Report, Chapter 3, Table 3. We calculate $s = A/B$ where A is the total assets of mutual funds, insurance companies, hedge funds, and broker-dealers and B is the total assets of banks and credit unions.

B.4 Parameterization: policy smoothing [Section 3.2]

In the estimation, we find a policy inertia coefficient of $\rho = 0.856$, suggesting that policy smoothing is an important feature of the data. As the strength of the signalling channel of negative interest rates will depend sensitively on the degree of policy inertia, we support the results of this estimation with further evidence, and—as for the reserve-to-deposit ratio—show sensitivity results in Section 3.4 in the main text.

Literature Figure B2(a) documents estimates of policy smoothing from the literature for the US, euro area, and four additional countries. Two key messages emerge. First, there is robust evidence for a large inertial component of monetary policy, irrespective of the estimation technique or country considered. Second, the estimates range from 0.80 (Primiceri et al., 2006, US) to 0.96 (Smets and Wouters, 2003, euro area). Thus, our baseline value of $\rho = 0.856$ is, if anything, on the more conservative side of possible parameterizations in terms of quantifying the strength of the signalling channel.[30]

Negative rates in Sweden There might be a concern that that these estimates are limited to periods in which policy rates were in positive territory. Figure B2(b) provides suggestive evidence from Sweden that policy inertia extends to negative rate episodes as well. Between February 2015 and February 2016, the Swedish Riksbank lowered the repo rate, its key policy rate, in four steps from 0% to −0.5%. Repo rate forecasts published by the Riksbank around the respective monetary policy decisions show that every negative rate decision came with a substantial downward revision of the forecasted path of the future policy rate, both extending the expected ZLB duration and lowering the expected future policy rate. This is consistent with inertial policy-setting as documented above.

[30] Rudebusch (2002, 2006) argues that observed policy inertia may, in fact, reflect persistent shocks rather than interest rate smoothing. However, recent work by Coibion and Gorodnichenko (2012) finds strong evidence in favour of the interest rate smoothing explanation.

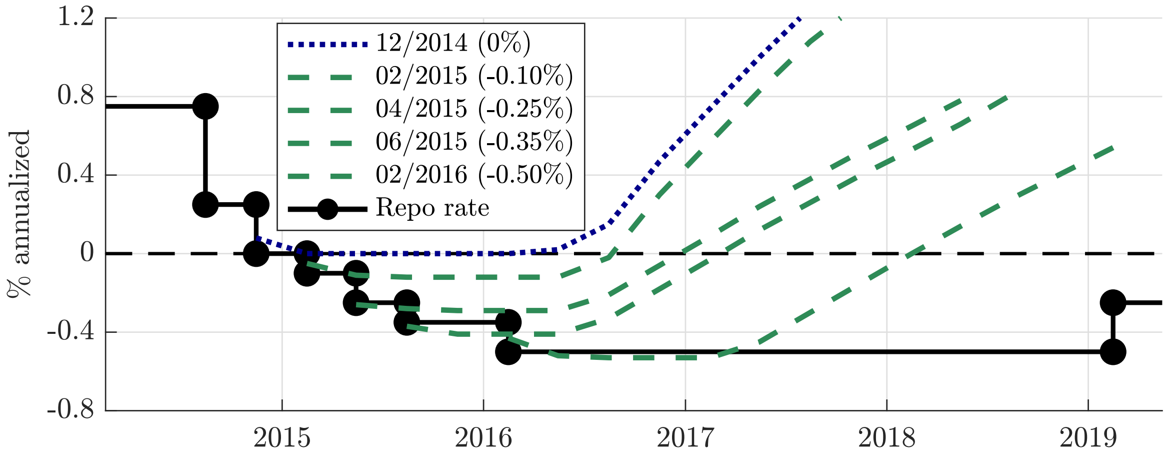
Figure B2: Monetary policy inertia in the literature and in practice

(a) Estimates of policy rule inertia

United States		**Euro area**	
Primiceri et al. (2006)	0.80	Smets and Wouters (2003)	0.96
Smets and Wouters (2007)	0.81	Christiano et al. (2010)	0.84
Coibion and Gorodnichenko (2012)	0.83	Darracq Pariès et al. (2011)	0.84
Brayton et al. (2014)	0.85	Coenen et al. (2018)	0.93
Christiano et al. (2014)	0.85	**Japan**	
United Kingdom		Sugo and Ueda (2007)	0.84
Burgess et al. (2013)	0.83	**Sweden**	
Switzerland		Adolfson et al. (2008)	0.88
Rudolf and Zurlinden (2014)	0.90	Christiano et al. (2011)	0.82

Note: Estimates of ρ for a selection of papers and central bank policy models. Brayton et al. (2014) is the Federal Reserve's FRB/US model, Burgess et al. (2013) is the Bank of England's COMPASS model, and Coenen et al. (2018) is the ECB's New Area Wide Model II.

(b) Riksbank repo rate forecasts during negative interest rates

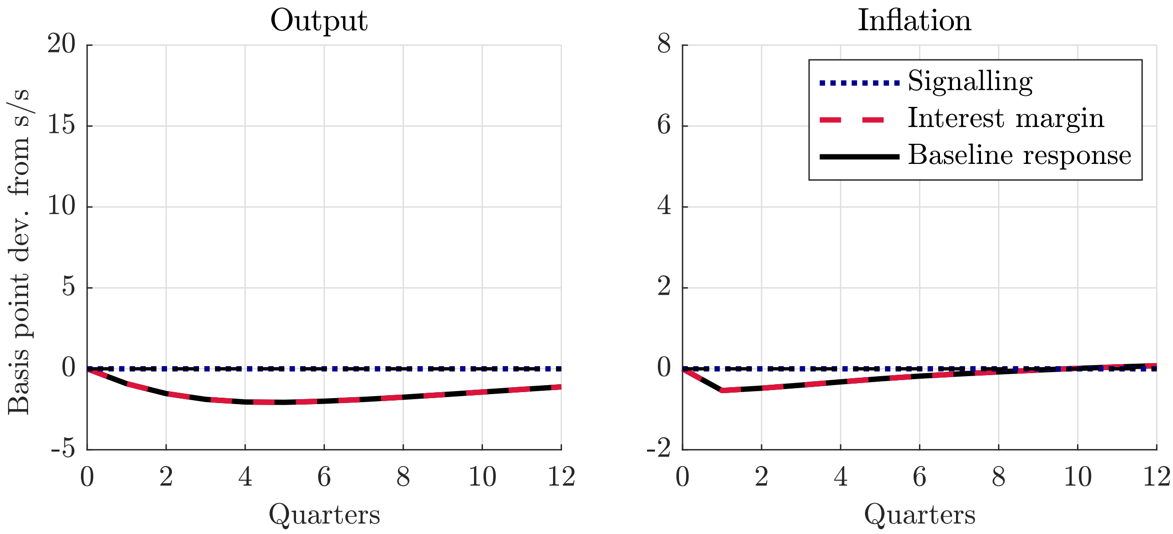


Note: The blue-dot and green-dash lines show the Riksbank's own repo rate forecasts around monetary policy meetings in which they lowered the repo rate, based on quarterly averages. The actual repo rate (black-solid line) is based on daily data. Source: Riksbank monetary policy reports.

B.5 Results: signalling channel without inertia [Section 3.3]

Figure B3 is analogous to Figure 6 in the main text and shows the signalling channel vs. net interest margin channel decomposition when $\rho = 0$ in the quantitative model. The figure shows that in the absence of r_{t-1} in the monetary policy rule, the signalling channel is completely shut down. This is true despite the existence of a range of further endogenous state variables in the model.

Figure B3: Contribution of signalling and interest margin channels (**no inertia**)



Note: Replication of Figure 6 without policy inertia ($\rho = 0$). Impulse responses to a -25bp iid monetary policy shock at the ZLB. Inflation is annualized. We linearly decompose the baseline response into "Signalling"—$\alpha = 0$ and $\rho = 0$, i.e. no costly interest margin channel—and "Interest margin"—difference between the baseline and "Signalling".

B.6 Results: derivation of bank profit decomposition [Section 3.3]

Baseline This section derives Equation (30) in the main text. From Equations (27) and (28), the evolution of net worth (conditional on not exiting) is given by

$$N_t = \left(R_{k,t}\Phi_{t-1} - \frac{R_{d,t-1} - \alpha\left(x_t\right) R_{t-1}}{\left(1-\alpha\left(x_t\right)\right)\Pi_t}\left(\Phi_{t-1} - 1\right)\right) N_{t-1}. \tag{B39}$$

Defining profits as $\texttt{prof}_t \equiv \Pi_t N_t / N_{t-1}$ and rearranging terms gives

$$\texttt{prof}_t = \left(\Pi_t R_{k,t} - R_{d,t-1}\right)\Phi_{t-1} + R_{d,t-1} - \frac{\alpha\left(x_t\right)}{1-\alpha\left(x_t\right)}\left(R_{d,t-1} - R_{t-1}\right)\left(\Phi_{t-1} - 1\right). \tag{B40}$$

Adding and subtracting $\mathbb{E}_{t-1}\Pi_t R_{k,t}\Phi_{t-1}$ gives

$$\begin{aligned}\texttt{prof}_t = &\left(\Pi_t \frac{\texttt{mpk}_t + Q_t - \delta}{Q_{t-1}} - \mathbb{E}_{t-1}\Pi_t \frac{\texttt{mpk}_t + Q_t - \delta}{Q_{t-1}}\right)\Phi_{t-1} \\ &+ \texttt{cs}_{t-1}\Phi_{t-1} + R_{d,t-1} - \frac{\alpha\left(x_t\right)}{1-\alpha\left(x_t\right)}\left(R_{d,t-1} - R_{t-1}\right)\left(\Phi_{t-1} - 1\right),\end{aligned} \tag{B41}$$

where $R_{k,t} = \frac{\texttt{mpk}_t + Q_t - \delta}{Q_{t-1}}$ and $\texttt{cs}_t \equiv \mathbb{E}_t \Pi_{t+1} R_{k,t+1} - R_{d,t}$.
Log-linearizing and collecting terms we arrive at Equation (30) in the main text.

Model with firm equity and loan finance Suppose instead that firms borrow from banks using a combination of equity and loans in proportion $\texttt{s}$ and $1-\texttt{s}$, respectively. In particular, suppose that the return to a banker on a unit of capital is given by

$$R_{s,t} = \texttt{s} R_{k,t} + \left(1-\texttt{s}\right) R_{l,t-1}, \tag{B42}$$

where $R_{l,t} \equiv \mathbb{E}_t R_{k,t+1}$. In this case, the credit spread is $\texttt{cs}_t \equiv \mathbb{E}_t \Pi_{t+1} R_{s,t+1} - R_{d,t}$ and the first-term on the right-hand side of Equation (B40) becomes $\left(\Pi_t R_{s,t} - R_{d,t-1}\right)\Phi_{t-1}$.

Adding and subtracting $\mathbb{E}_{t-1}\Pi_t R_{s,t}$ gives

$$\left(\Pi_t R_{s,t} - \mathbb{E}_{t-1}\Pi_t R_{s,t}\right)\Phi_{t-1} + \texttt{cs}_{t-1}\Phi_{t-1}. \tag{B43}$$

Log-linearizing around the deterministic steady state gives

$$R_s\Phi\left(\hat{\pi}_t - \mathbb{E}_{t-1}\hat{\pi}_t\right) + R_s\Phi\left(\hat{r}_{s,t} - \mathbb{E}_{t-1}\hat{r}_{s,t}\right) + \texttt{cs}\Phi\left(\hat{\texttt{cs}}_{t-1} + \hat{\phi}_{t-1}\right). \tag{B44}$$

while log-linearizing $R_{s,t}$ gives

$$R_s\hat{r}_{s,t} = \texttt{s}\left(\texttt{mpk}\cdot\hat{\texttt{mpk}}_t + \hat{q}_t\right) + \left(1-\texttt{s}\right)\mathbb{E}_{t-1}\left(\texttt{mpk}\cdot\hat{\texttt{mpk}}_t + \hat{q}_t\right) - R_k\hat{q}_{t-1}. \tag{B45}$$

Therefore

$$R_s\left(\hat{r}_{s,t}-\mathbb{E}_{t-1}r_{s,t}\right)=\mathtt{s}\cdot\mathtt{mpk}\left(\hat{\mathtt{mpk}}_t-\mathbb{E}_{t-1}\hat{\mathtt{mpk}}_t\right)+\mathtt{s}\left(\hat{q}_t-\mathbb{E}_{t-1}\hat{q}_t\right). \tag{B46}$$

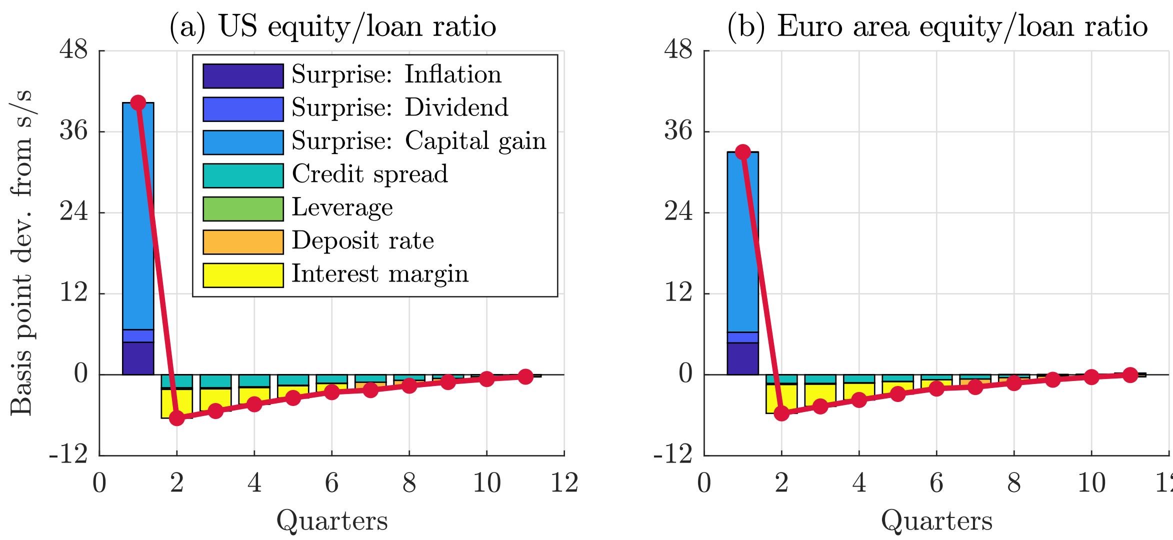
Finally, the augmented version of Equation (30) that accounts for bank assets being composed of a mix of equity and loans is given by

$$\begin{aligned}\hat{\mathtt{prof}}_t=&\underbrace{\frac{R_k\Phi}{\mathtt{prof}}\left(\hat{\pi}_t-\mathbb{E}_{t-1}\hat{\pi}_t\right)}_{\text{Surprise: Inflation}}+\underbrace{\mathtt{s}\frac{\mathtt{mpk}\Phi}{\mathtt{prof}}\left(\hat{\mathtt{mpk}}_t-\mathbb{E}_{t-1}\hat{\mathtt{mpk}}_t\right)}_{\text{Surprise: Dividend}}+\underbrace{\mathtt{s}\frac{\Phi}{\mathtt{prof}}\left(\hat{q}_t-\mathbb{E}_{t-1}\hat{q}_t\right)}_{\text{Surprise: Capital gain}}\\&+\underbrace{\frac{\mathtt{cs}\Phi}{\mathtt{prof}}\hat{\mathtt{cs}}_{t-1}}_{\text{Credit spread}}+\underbrace{\frac{\mathtt{cs}\Phi}{\mathtt{prof}}\hat{\phi}_{t-1}}_{\text{Leverage}}+\underbrace{\frac{R_d}{\mathtt{prof}}\hat{r}_{d,t-1}}_{\text{Deposit rate}}-\underbrace{\frac{\alpha}{1-\alpha}\frac{R_d\left(\Phi-1\right)}{\mathtt{prof}}\left(\hat{r}_{d,t-1}-\hat{r}_{t-1}\right)}_{\text{Interest margin channel}}.\end{aligned} \tag{B47}$$

When $\mathtt{s}=1$, the formulation is the same as Equation (30) in the main text.

Based on De Fiore and Uhlig (2011) though, the debt to equity ratio of the non-financial sector is 0.43 in the US and 0.64 in the euro area. This translates to $\mathtt{s}_{US}=1/1.43\approx 0.70$ and $\mathtt{s}_{EA}=1/1.64\approx 0.61$, respectively. Figure B4 supplements our analysis on the robustness of our model to changes in the firm equity-to-loan ratio showing the results of the bank profit decomposition in Figure 6 in the main text for $\mathtt{s}_{US}$ and $\mathtt{s}_{EA}$.

Figure B4: Decomposition of bank profits
— Sensitivity with respect to **equity/loan ratio** —



Note: Replication of Figure 7 for alternative firm equity/loan ratios $\mathtt{s}$. (a) $\mathtt{s}_{US}=0.70$, (b) $\mathtt{s}_{EA}=0.61$. $\alpha=0.2$, $\rho=0.85$. The red-dot line plots the impulse response of bank profits to a -25bp iid monetary policy shock at the ZLB. Stacked bars decompose the impulse response for every period.

B.7 Results: further robustness [Section 3.4]

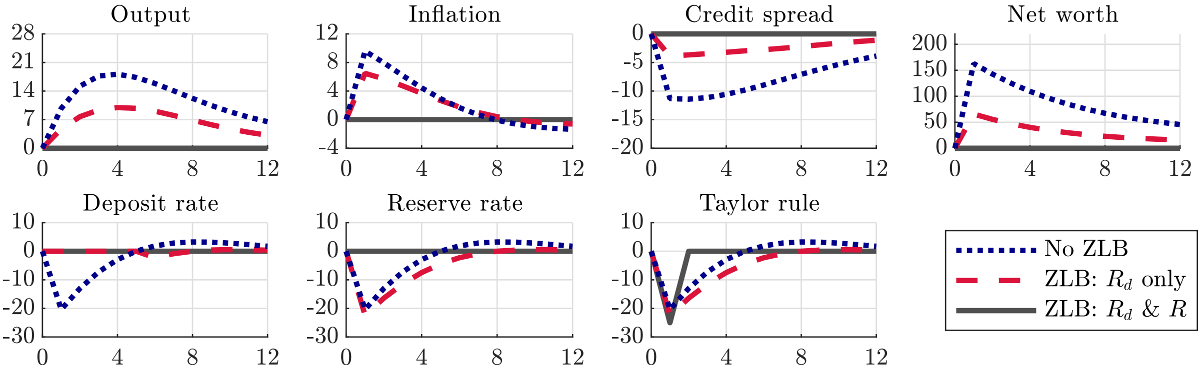
This section provides additional details and reports the results for the further robustness exercises summarized towards the end of Section 3.4 in the main text.

Sensitivity w.r.t. Frisch elasticity As Chetty et al. (2011) report, macro estimates of the Frisch elasticity from real business cycle models range from 2.61 to 4. Our baseline value of 3.75 is within this range. Smets and Wouters (2007) estimate the value to be 1.92 whereas micro estimates are around 0.82. Another popular choice in the literature for calibrated models is to set the elasticity to 1, between the micro and macro estimates as in Hazell et al. (2022). Figure B5 replicates Figure 5 in the main text for a range of plausible empirical values of the inverse elasticity of labor supply/ Frisch elasticity φ. The figure shows that our results regarding the effectiveness of negative interest rates are robust to changes of the exact value of φ. A higher inverse Frisch elasticity (i.e. a lower labor supply elasticity of the household in the model) reduces the expansion in output in response to a monetary policy easing relative to the baseline (depicted in rows 5 and 6). However, since this is true for both monetary policy surprises in normal times and at the ZLB, the relative efficiency of a monetary policy easing into negative territory remains broadly unchanged (as can be seen comparing the red and blue lines across specifications).

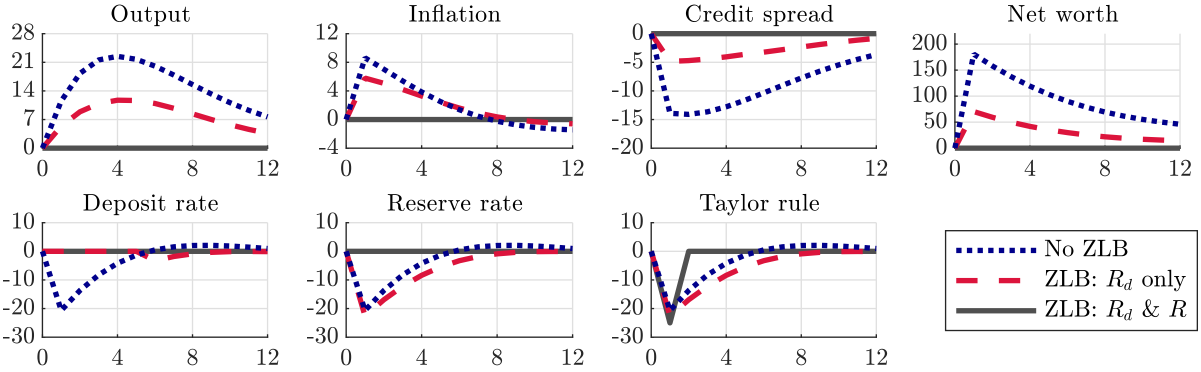
Sensitivity w.r.t. Phillips curve slope As Harding et al. (2022) report, estimates of the new-Keynesian Phillips curve slope in the literature range from 0.009 to 0.014. Hazell et al. (2022) estimate the unemployment-inflation slope to be 0.0062. Based on a Frisch elasticity between 1 and 3.62 (as above), this gives a Phillips curve slope in the range $0.006 - 0.023$. With a Calvo parameter of 0.9, our baseline Phillips curve slope is 0.012, well within both ranges. Figure B6 replicates Figure 5 in the main text for different combinations of plausible empirical values of the inverse elasticity of labor supply/ Frisch elasticity φ and the Calvo parameter ι keeping the unemployment-inflation slope constant at 0.0062 as suggested by Hazell et al. (2022). The figure shows that our results regarding the effectiveness of negative interest rates are robust to changes in the slope of the new-Keynesian Phillips Curve. A higher inverse Frisch elasticity paired with tighter price rigidity (i.e. a larger Calvo parameter) reduces both the expansion in output and inflation in response to a monetary policy easing relative to the baseline case. However—as in the case where we just vary the Frisch elasticity φ—since this is true for monetary policy surprises in normal times and at the ZLB, the relative efficiency of a monetary policy easing into negative territory remains broadly unchanged.

Figure B5: Monetary policy shock with inertia in the policy rule
— Sensitivity with respect to **inverse Frisch elasticity** —

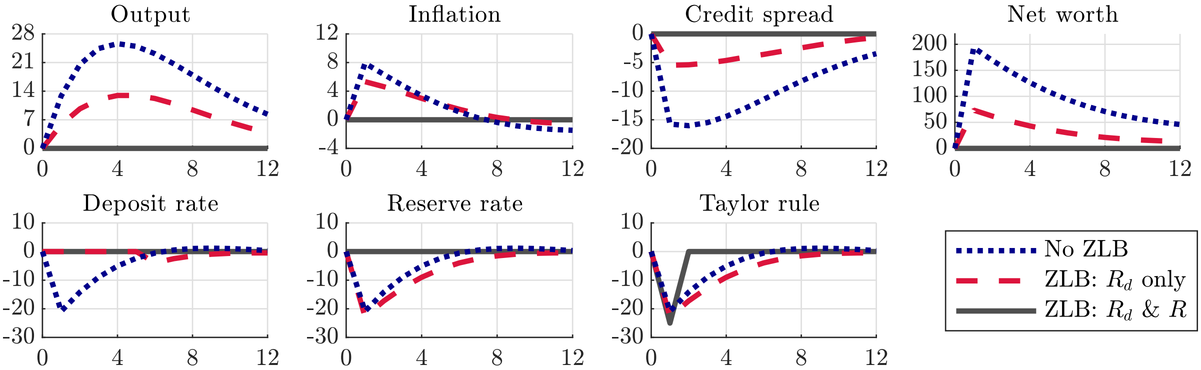
(a) High inverse Frisch elasticity



(b) Medium inverse Frisch elasticity



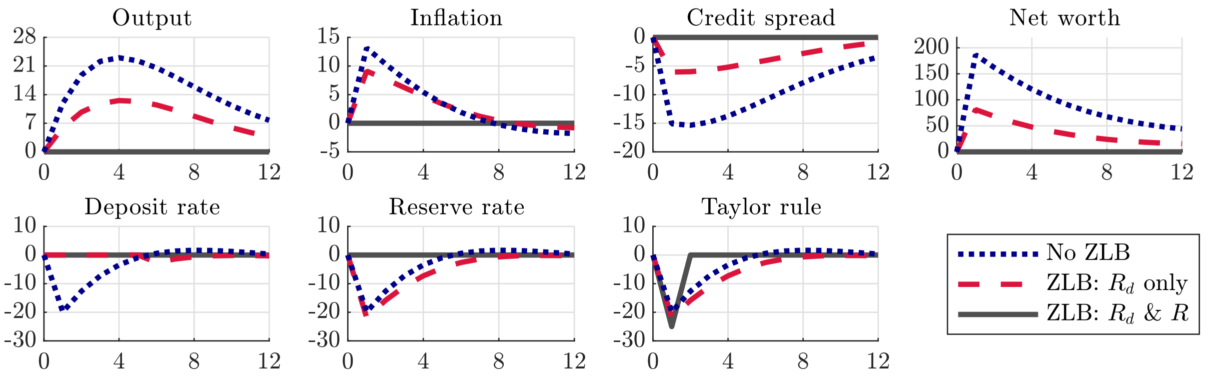
(c) Baseline inverse Frisch elasticity



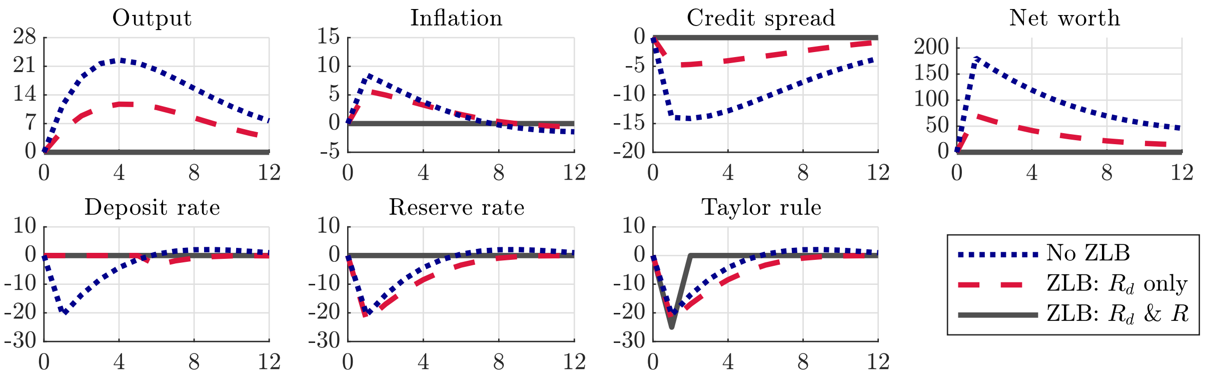
NOTE: $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a -25bp iid monetary policy shock at the ZLB. Rows 1 and 2 show results for an inverse Frisch elasticity of $\varphi = 1$ (Hazell et al., 2022), rows 3 and 4 for $\varphi = 0.521$ (Smets and Wouters, 2007), and rows 5 and 6 for $\varphi = 0.276$ (baseline). Interest rates are in annualized basis points. All other variables are in basis point deviation from steady state. Inflation is annualized.

Figure B6: Monetary policy shock with inertia in the policy rule
— Sensitivity with respect to **Phillips Curve slope** —

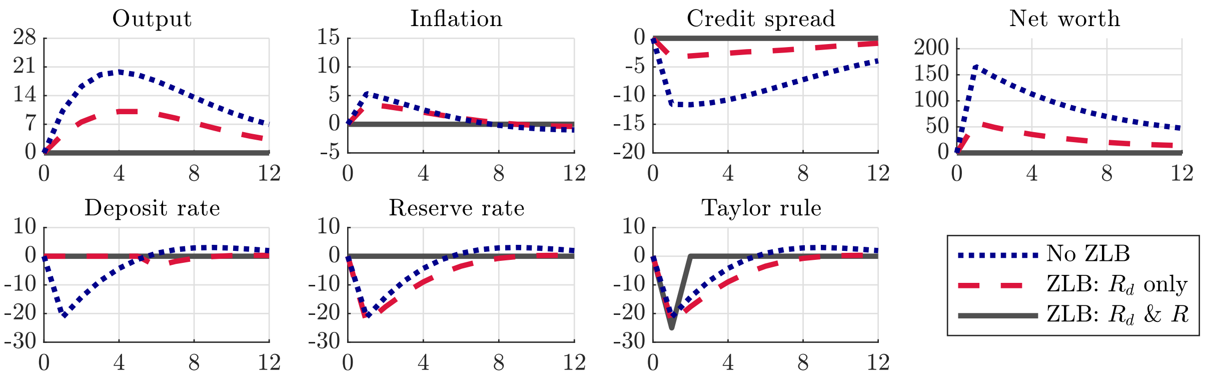
(a) Steep Phillips Curve



(b) Medium Phillips Curve



(c) Flat Phillips Curve



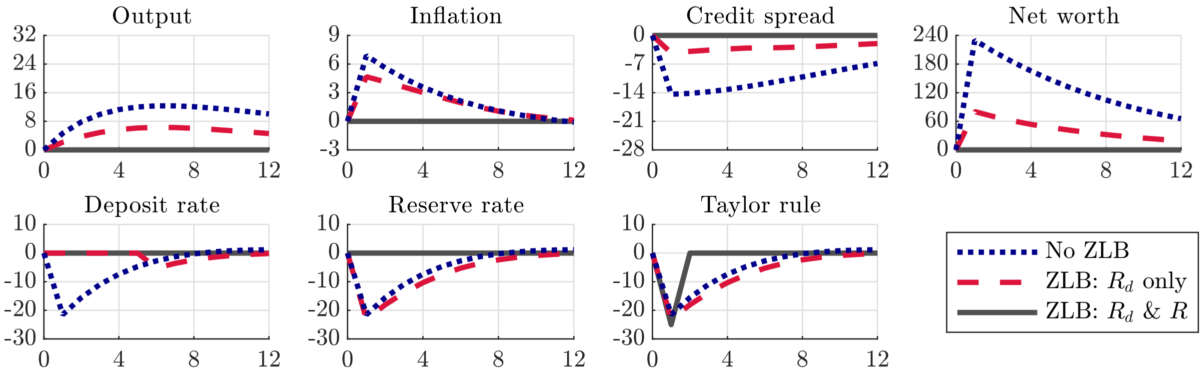
NOTE: $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a -25bp iid monetary policy shock at the ZLB. The values for the inverse Frisch elasticity φ from Figure B5 are paired with a Calvo parameter ι such that the unemployment-inflation trade-off in all specifications is 0.0062 as suggested by Hazell et al. (2022). Rows 1 and 2 depict results for $\{\varphi = 0.276, \iota = 0.865\}$, rows 3 and 4 for $\{\varphi = 0.521, \iota = 0.901\}$, and rows 5 and 6 for $\{\varphi = 1, \iota = 0.929\}$. Interest rates are in annualized basis points. All other variables are in basis point deviation from steady state. Inflation is annualized.

Sensitivity w.r.t. investment elasticity The inverse investment elasticity η is difficult to pin down from the literature which is why we include the parameter in the estimation matching a range of empirical moments (see Appendix B.3 for details). Our estimate of $\eta = 1.617$ is close to the 1.728 value picked by Gertler and Karadi (2011) and delivers a net worth response to an unconstrained 25bp monetary policy shock in line with the empirical response in Jarociński and Karadi (2020) (194bp versus 210bp on impact). Since the net worth response to monetary policy was not targeted in our estimation, this outcome provides external validation for our parameterization. However, as the parameter is crucial for the strength of the financial accelerator, we test the robustness of our main results for a wide parameter range in Figure B7. The figure shows that our results regarding the effectiveness of negative interest rates are robust to changes in the investment elasticity. Increasing the investment elasticity strengthens the impact response but decreases the persistence of monetary policy. Figure B8 replicates Figure 7 in the main text. In terms of banks profitability, increasing the investment elasticity decreases windfall capital gains for banks (as asset prices are less responsive) but raises windfall dividends (as investment is more responsive) to a negative rate shock.

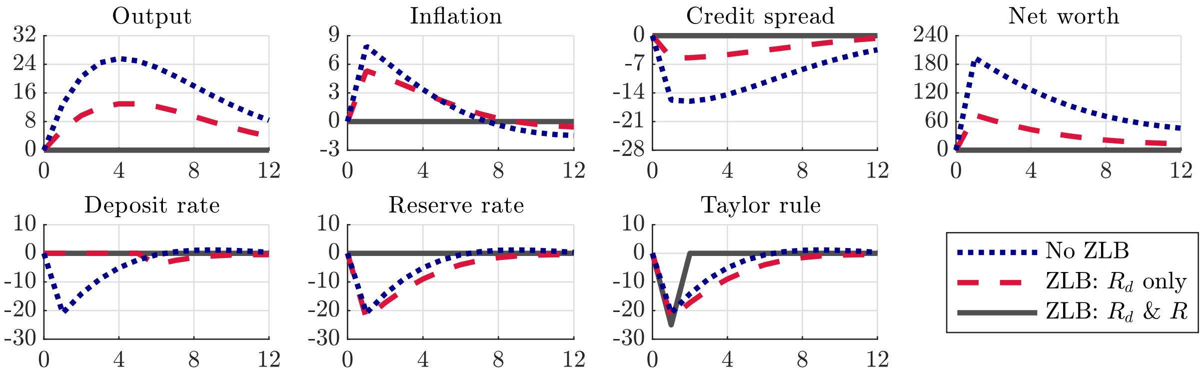
Sensitivity w.r.t. wage rigidities Finally, we augment the model with nominal wage rigidities. Figures B9-B11 replicate Figures 4-6 in the main text when nominal wage rigidities supplement nominal price rigidity in the model. The extension of the model in this dimension is straightforward. Appendix B.8 details the necessary modifications. For simplicity, we keep the baseline calibration unchanged and set the structural parameters associated with Calvo wage rigidities as follows: ϵ_w—the elasticity of substitution between different types of labor—is set to 4.167 (equal to the value picked for price rigidities), and ι_w—the probability of not being able to adjust wages next period—is set to 0.5. We also marginally increase the size of the risk premium shock that takes the model to the ZLB in order to regenerate our baseline experiment with the ZLB binding for four periods (intuitively, this adjustment is needed due to the additional persistence in the model). The figures show that a high Frisch elasticity and nominal wage rigidities are, to some degree, substitutable in our analysis and that our results regarding the effectiveness of negative interest rates are robust to the introduction of rigid wages. The introduction of an additional nominal rigidity makes monetary policy interventions more powerful. However—as in the case where we vary the Frisch elasticity and the slope of the new-Keynesian Phillips Curve—since this is true for monetary policy surprises in normal times and at the ZLB, the relative efficiency of a monetary policy easing into negative territory remains broadly unchanged. Similar results can be obtained for alternative specifications of ϵ_w and ι_w.

Figure B7: Monetary policy shock with inertia in the policy rule
— Sensitivity with respect to **inverse investment elasticity** —

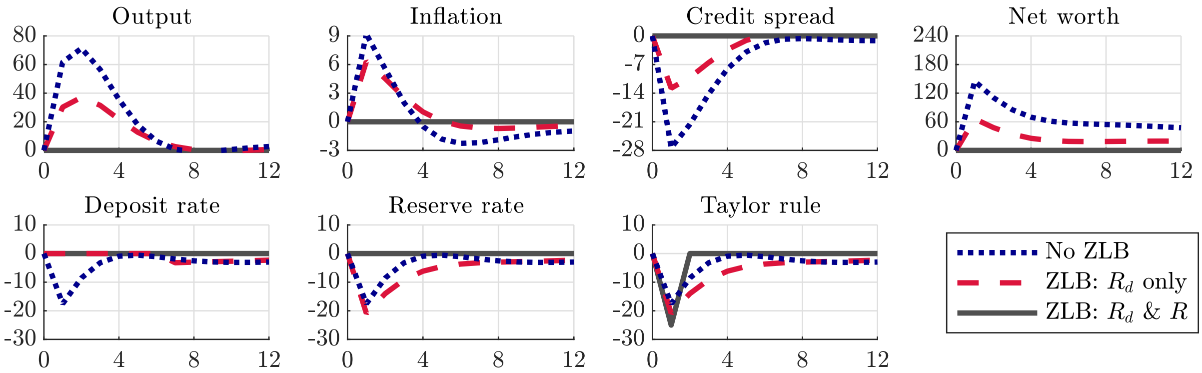
(a) High inverse investment elasticity



(b) Baseline inverse investment elasticity

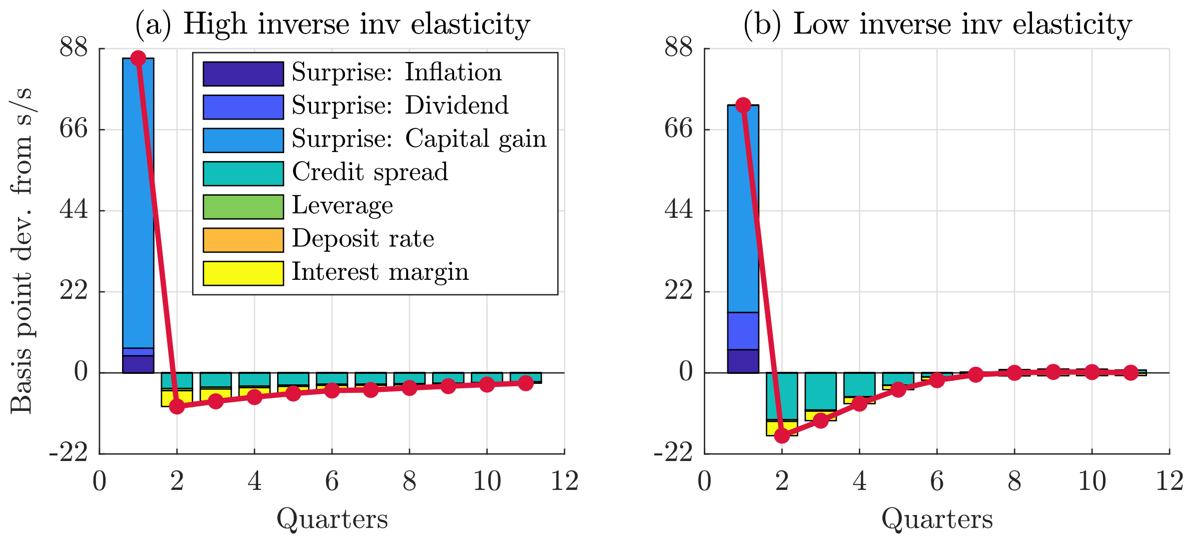


(c) Low inverse investment elasticity



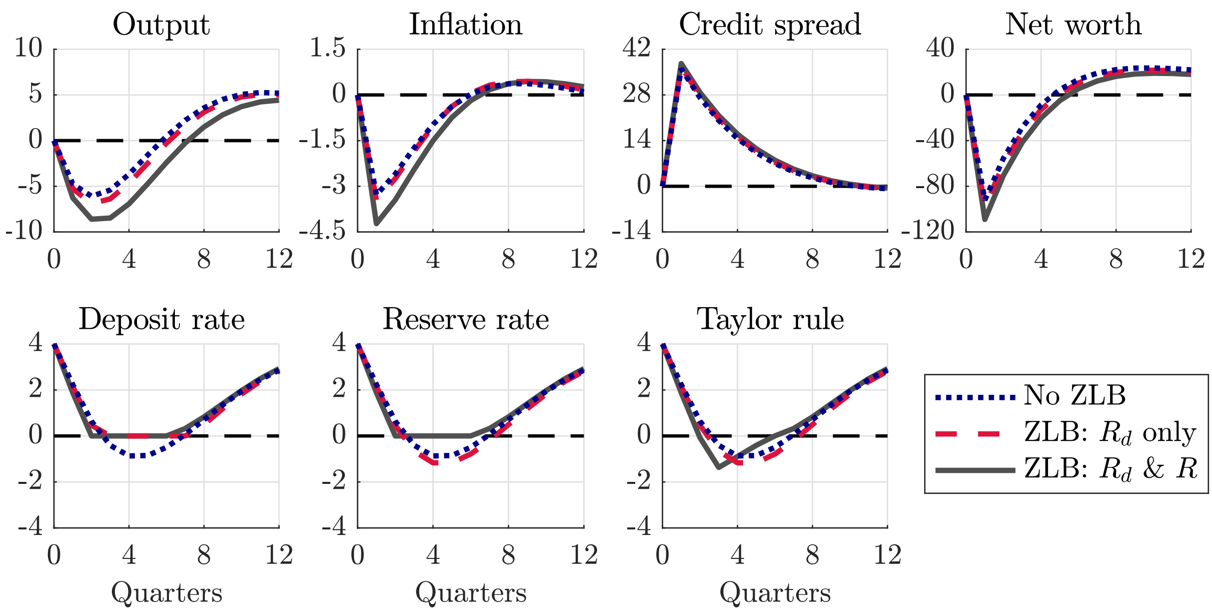
Note: $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a -25bp iid monetary policy shock at the ZLB. Rows 1 and 2 show results for an inverse investment elasticity of $\eta = 10$, rows 3 and 4 for $\eta = 1.617$ (baseline), and rows 5 and 6 for $\eta = 0.1$. Interest rates are in annualized basis points. All other variables are in basis point deviation from steady state. Inflation is annualized.

Figure B8: Decomposition of bank profits
— Sensitivity with respect to **inverse investment elasticity** —



Note: Replication of Figure 7 for alternative inverse investment elasticities, η. (a) $\eta = 10$, (b) $\eta = 0.1$. $\alpha = 0.2$, $\rho = 0.85$. The red-dot line plots the impulse response of bank profits to a -25bp iid monetary policy shock at the ZLB. Stacked bars decompose the impulse response for every period.

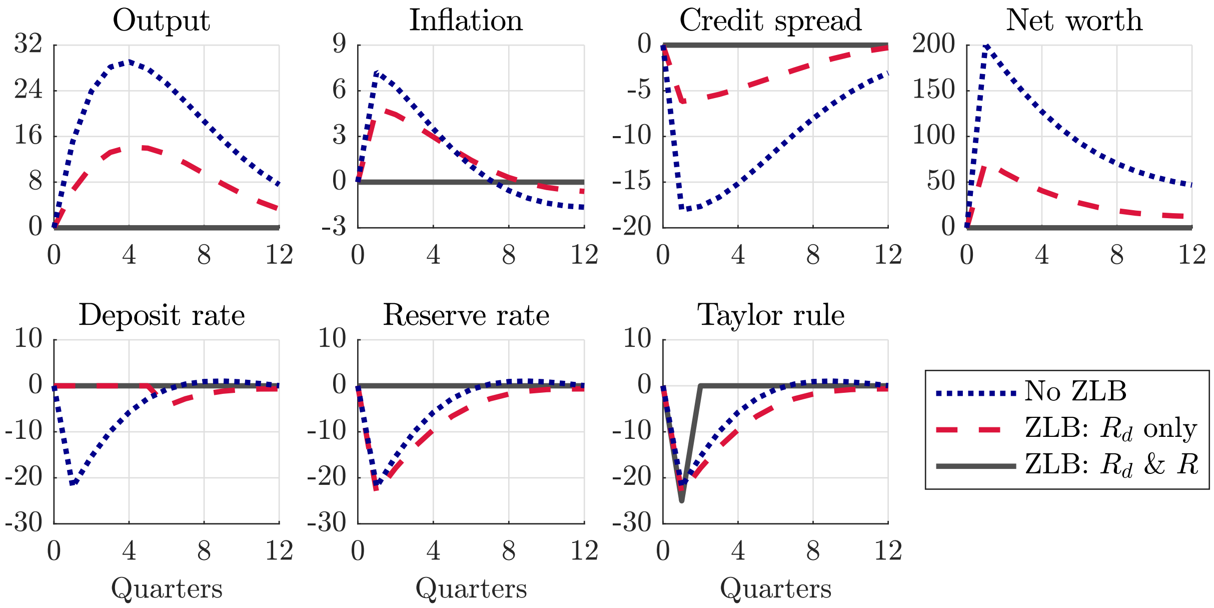
Figure B9: Wage rigidities: Risk premium shock with inertia in the policy rule



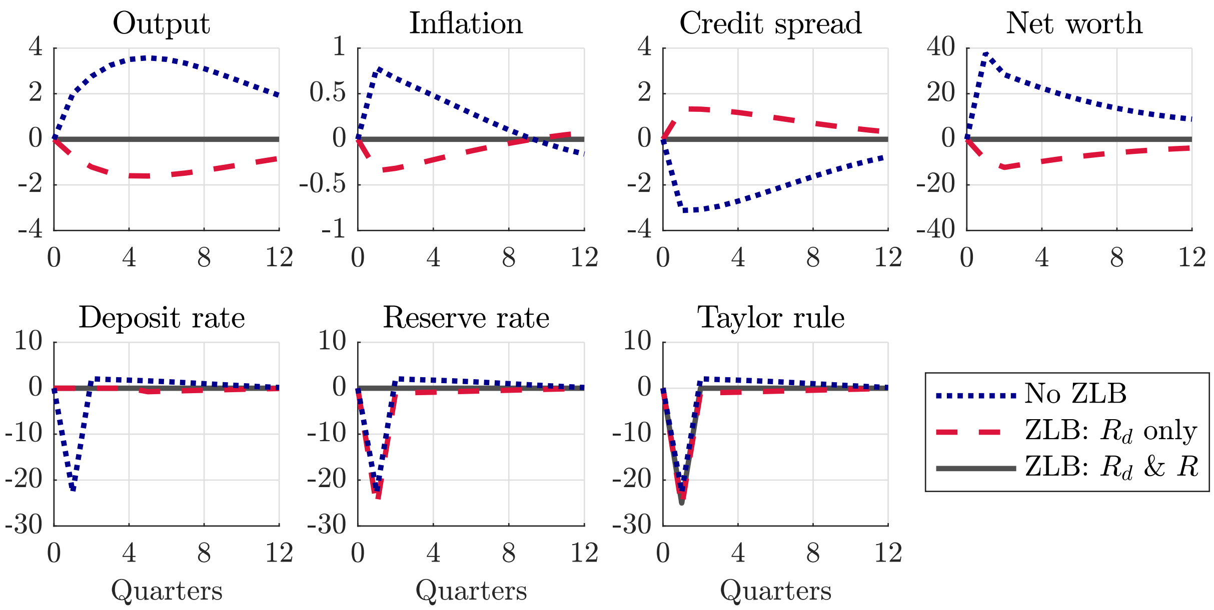
Note: Replication of Figure 4 with wage rigidities added to model ($\iota_w = 0.5$). $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a risk premium shock that brings the economy to the ZLB for 4 quarters. All interest rates displayed are in annualized percent. All other variables are in 100×log-deviation from steady state. Inflation is annualized.

Figure B10: Wage rigidities: Monetary policy shock in negative territory

(a) Policy rule with inertia

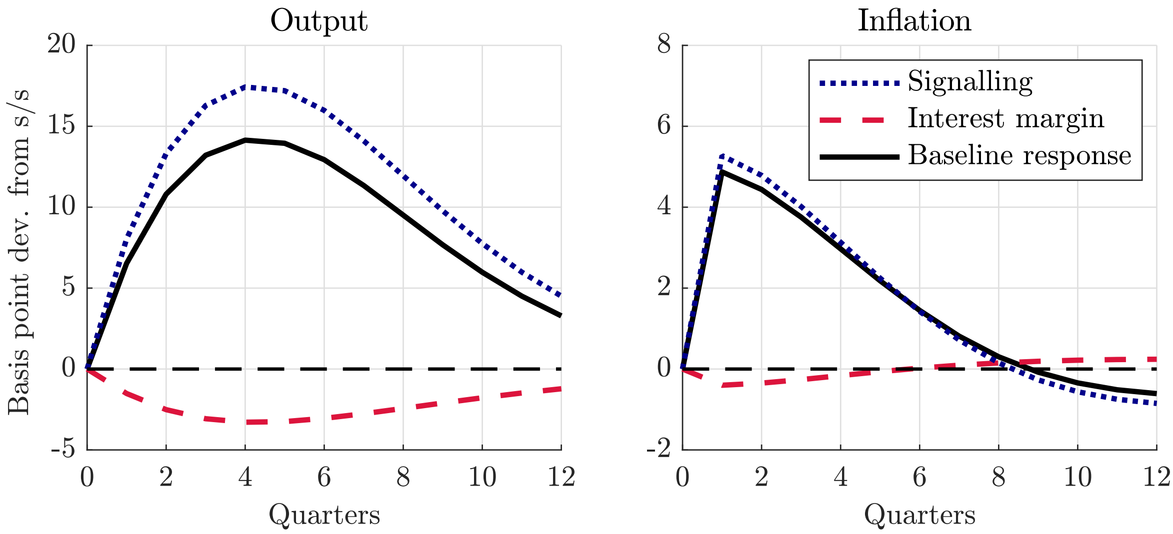


(b) Policy rule without inertia



Note: Replication of Figure 5 with wage rigidities added to model ($\iota_w = 0.5$). (a) $\alpha = 0.2$ and $\rho = 0.85$, (b)$\alpha = 0.2$ and $\rho = 0$. Impulse responses to a -25bp iid monetary policy shock at the ZLB. Interest rates are in annualized basis points. All other variables are in basis point deviation from steady state. Inflation is annualized.

Figure B11: Wage rigidities: Contribution of signalling and interest margin channels



Note: Replication of Figure 6 with wage rigidities added to model ($\iota_w = 0.5$). Impulse responses to a −25bp iid monetary policy shock at the ZLB. Inflation is annualized. We linearly decompose the baseline response into "Signalling"—$\alpha = 0$ and $\rho = 0.85$, i.e. no costly interest margin channel—and "Interest margin"—difference between the baseline and "Signalling".

B.8 Results: equilibrium with wage rigidities [Section 3.4]

We add nominal wage rigidities to the quantitative model following Erceg et al. (2000). Households supply homogeneous labor $L_{h,t}$ at price $W_{h,t}$. Monopolistic labor unions, owned by households, diversify and sell the labor good to intermediate goods firms as CES aggregate L_t at mark-up price W_t. In equilibrium, this extends the model to 27 equations in 27 endogenous variables, $\{Y_t,\ Y_{m,t},\ L_t,\ L_{h,t},\ C_t,\ \tilde{C}_t,\ \Lambda_{t,t+1},\ \mu_t,\ K_t,\ I_t,\ I_{n,t},\ N_t,\ \Phi_t,\ \Delta_t,\ \Delta_{w,t},\ W_t,\ W_{h,t},\ \Pi_t,\ \Pi_{w,t},\ X_t,\ P_{m,t},\ Q_t,\ R_{k,t},\ R_{T,t},\ R_t,\ R_{d,t},\ CS_t\}$, and 3 exogenous processes, $\{\zeta_t,\ \epsilon_t,\ \varepsilon_{m,t}\}$. In the following we only restate the equations that are new or modified relative to the overview in B.2.

Households

- Labor supply (modified)

$$\mu_t W_{h,t} = \chi L_{h,t}^{\varphi} \tag{B48}$$

Labor Unions

- Wage Phillips Curve (new)

$$\left(\frac{\epsilon_w}{\epsilon_w - 1}\right)\frac{\mathcal{D}_{w,t}}{\mathcal{F}_{w,t}} = \left[\frac{1 - \iota_w \left(\Pi_t\, \Pi_{w,t}\right)^{\epsilon_w - 1}}{1 - \iota_w}\right]^{\frac{1}{1-\epsilon_w}} \tag{B49}$$

$$\text{where} \quad \mathcal{D}_{w,t} \equiv \mu_t W_{h,t} L_t + \beta \iota_w \mathbb{E}_t \left(\Pi_{t+1}\, \Pi_{wt+1}\right)^{\epsilon_w} \mathcal{D}_{w,t+1},$$
$$\mathcal{F}_{w,t} \equiv \mu_t W_t L_t + \beta \iota_w \mathbb{E}_t \left(\Pi_{t+1}\, \Pi_{wt+1}\right)^{\epsilon_w - 1} \mathcal{F}_{w,t+1}.$$

- Wage dispersion (new)

$$\Delta_{w,t} = (1 - \iota_w)\left[\left(\frac{\epsilon_w}{\epsilon_w - 1}\right)\frac{\mathcal{D}_{w,t}}{\mathcal{F}_{w,t}}\right]^{-\epsilon_w} + \iota_w \left(\Pi_t\, \Pi_{w,t}\right)^{\epsilon_w} \Delta_{w,t-1} \tag{B50}$$

- Wage inflation (new)

$$\Pi_{w,t} = W_t / W_{t-1} \tag{B51}$$

General equilibrium

- Aggregate labor (new)

$$L_t = L_{h,t} / \Delta_{w,t}, \tag{B52}$$

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