— Appendix —

The Signalling Channel of Negative Interest Rates

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A Stylized model and optimal policy

Appendix A relates to Section 2 (main paper) on optimal policy in the stylized model. Section A.1 derives a simple model of reserve demand. Section A.2 documents the full derivation of the stylized model. Section A.3 shows that the stylized model captures key features of the quantitative model if a Taylor-type policy rule is added. Section A.4 derives the first-order conditions under commitment and discretion and proves Propositions 2 and 3, respectively. Section A.5 shows that not any private-sector state variable makes negative rates optimal. Section A.6 describes the non-linear solution algorithm used to generate our numerical results. Section A.7 derives the consumption equivalent measure of welfare and provides welfare results. Section A.8 documents one additional optimal policy experiment. Finally, Section A.9 derives the analytical solutions for a simplified version of the model used for comparative statics in the main text.[1]

[1] For expositional clarity, we simplify the notation compared to Section 2. In particular, we drop time subscripts and replace them with recursive notation. y denotes the output gap.

A.1 Set up: A simple model of reserve demand [Section 2.1]

We analyse a single bank's reserve demand decision. At $t = 0$, the bank has L loans and $D_r = L$ retail deposits (without loss of generality, we set $D_r = 1$). The bank also raises wholesale deposits, $D_w \geq 0$, and places them in its reserve account at the central bank to obtain $A = D_w$ reserves. At $t = 1$, loans are repaid at R_l, reserves are repaid at R, and all deposits $(D_r + D_w)$ are repaid at R_d. At $t = 2$, a fraction $\sigma \in (0, 1)$ of total deposits ($\tilde{D} = \sigma(1 + D_w)$) flow out of the bank with probability 1/2. The cost function $\frac{2\theta}{1+\xi}\left(\max\left[\tilde{D} - A, 0\right]\right)^{1+\xi}$ (for $\xi > 1$) captures interbank market frictions and the illiquidity of loans (reserves are perfectly liquid). The bank solves the following problem:

$$\max_A \mathbb{E}\left\{(R_l - R_d) + (R - R_d)A - \frac{2\theta}{1+\xi}\left(\max\left[\tilde{D} - A, 0\right]\right)^{1+\xi}\right\},$$
$$\max_A \left\{(R_l - R_d) + (R - R_d)A - \frac{\theta}{1+\xi}\left(\max\left[\sigma(1+A) - A, 0\right]\right)^{1+\xi}\right\}. \tag{A1}$$

The solution is as follows: If $R > R_d$, the demand for reserves is unbounded. If $R < R_d$, the bank will optimally chose a level of reserves such that a potential outflow of deposits is associated with non-zero cost (the left-hand side of the max operator). In this case, the optimal level of reserves, A^*, is given by

$$A^* = \frac{\sigma}{1-\sigma} - \frac{1}{1-\sigma}\left(\frac{R_d - R}{\theta(1-\sigma)}\right)^{1/\xi}. \tag{A2}$$

Optimal reserve holdings, A^*, are increasing in the level of liquidity risk, σ. When there is no liquidity risk, $\sigma = 0$, the bank holds no reserves. Optimal reserve holdings are also increasing in the illiquidity of loans, θ. When loans are fully liquid, $\theta = 0$, the bank holds no reserves. Defining $x \equiv R/R_d$ and the reserve-to-deposit ratio as $\alpha \equiv A/(1 + A)$, we can rewrite the demand curve, $\alpha(x)$, as

$$\alpha(x) = \frac{\sigma - \left(\frac{R_d(1-x)}{\theta(1-\sigma)}\right)^{1/\xi}}{1 - \left(\frac{R_d(1-x)}{\theta(1-\sigma)}\right)^{1/\xi}}. \tag{A3}$$

This demand curve has the following properties: $\alpha(1) = \sigma > 0$, $\alpha'(x) > 0$, and $\alpha''(x) > 0$.

A.2 Log-linear equilibrium: derivation [Section 2.2]

New-Keynesian IS equation The household problems and first-order conditions are given in the main text. In steady state, $R_d = 1/\beta$. The log-linear form of the first-order conditions for the saver household are given by

$$c_{s,t} = \mathbb{E}_t c_{s,t+1} - \frac{1}{\sigma}\left(r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t\right), \tag{A4}$$

$$\varphi l_{s,t} = -\sigma c_{s,t} + w_{s,t}, \tag{A5}$$

where lower case letters refer to log-levels. The borrower household's conditions are

$$c_{b,t} = \mathbb{E}_t c_{b,t+1} - \frac{1}{\sigma}\left(r_{b,t} - \mathbb{E}_t \pi_{t+1} - s_t\right), \tag{A6}$$

$$\varphi l_{b,t} = -\sigma c_{b,t} + w_{b,t}, \tag{A7}$$

where, in steady state, $R_b = 1/\beta_b$. The log-linear aggregate resource constraint is given by $y_t = (1 - \mathsf{c})\, c_{s,t} + \mathsf{c} c_{b,t}$, where $\mathsf{c} \equiv C_b/Y$. Combining this definition with the two individual Euler equations gives the aggregate Euler equation:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1-\mathsf{c}}{\sigma}\left(r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t\right) - \frac{\mathsf{c}}{\sigma}\left(\mathbb{E}_t r_{b,t+1} - \mathbb{E}_t \pi_{t+1} - s_t\right). \tag{A8}$$

Next, substituting the transfer from savers to borrowers into the borrower household's budget constraint gives the following simple borrower household consumption function: $C_{b,t} = B_t$. Using the definition for leverage, $\Phi_t = B_t/N_t$, the log-linear form of the borrower household consumption function is given by $c_{b,t} = \phi_t + n_t$. Rearranging the borrower household's Euler condition, $\frac{1}{\sigma}\left(r_{b,t} - \mathbb{E}_t \pi_{t+1} - s_t\right) = \mathbb{E}_t c_{b,t+1} - c_{b,t}$, and combining it with the consumption function above, we can rewrite the aggregate Euler equation as

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1-\mathsf{c}}{\sigma}\left(r_{d,t} - \mathbb{E}_t \pi_{t+1} - s_t\right) - \mathsf{c}\left(\mathbb{E}_t \phi_{t+1} - \phi_t + \mathbb{E}_t n_{t+1} - n_t\right). \tag{A9}$$

New-Keynesian Phillips curve Log-linearizing the production sector's first-order conditions yields the textbook new-Keynesian Phillips curve in terms of marginal cost,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\iota\beta)(1-\iota)}{\iota} mc_t. \tag{A10}$$

Log-linear marginal cost and aggregate output are given by $mc_t = \omega w_{s,t} + (1-\omega) w_{b,t}$ and $y_t = \omega l_{s,t} + (1-\omega) l_{b,t}$, respectively. Using the two labor-supply first-order conditions from the household problem, we can rewrite marginal cost as follows:

$$mc_t = (\varphi + \sigma) y_t, \tag{A11}$$

and the Phillips curve as

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\iota\beta)(1-\iota)(\varphi+\sigma)}{\iota} y_t. \tag{A12}$$

Note that since we only consider disturbances to households' subjective discount factors, the output gap coincides with output and hence y_t can be relabeled as the output gap.

Financial sector equilibrium conditions Steady state leverage is given by $\bar{N}$. The log-linear net worth evolution equation is given by

$$n_{t+1} = \theta R \left(n_t + \Phi (r_{b,t} - \pi_{t+1}) - (\Phi - 1) \left(\frac{r_{d,t} - \alpha r_t}{1-\alpha} - \pi_{t+1} \right) \right). \tag{A13}$$

When $\theta = 0$, then $n_{t+1} = 0$. The steady state tax on banks ensures that in steady state $R_b (1-\tau) = R_d$. The log-linear incentive compatibility constraint is given by

$$\phi_t = (\mathbb{E}_t m_{t,t+1} - \pi_{t+1}) + \theta \mathbb{E}_t \phi_{t+1} + \left(\Phi r_{b,t} - (\Phi - 1) \frac{r_{d,t} - \alpha r_t}{1-\alpha} \right). \tag{A14}$$

where $m_{t,t+1}$ is the log-linear stochastic discount factor of the saver household.

Substituting for $r_{b,t}$ using the borrower household's Euler equation gives

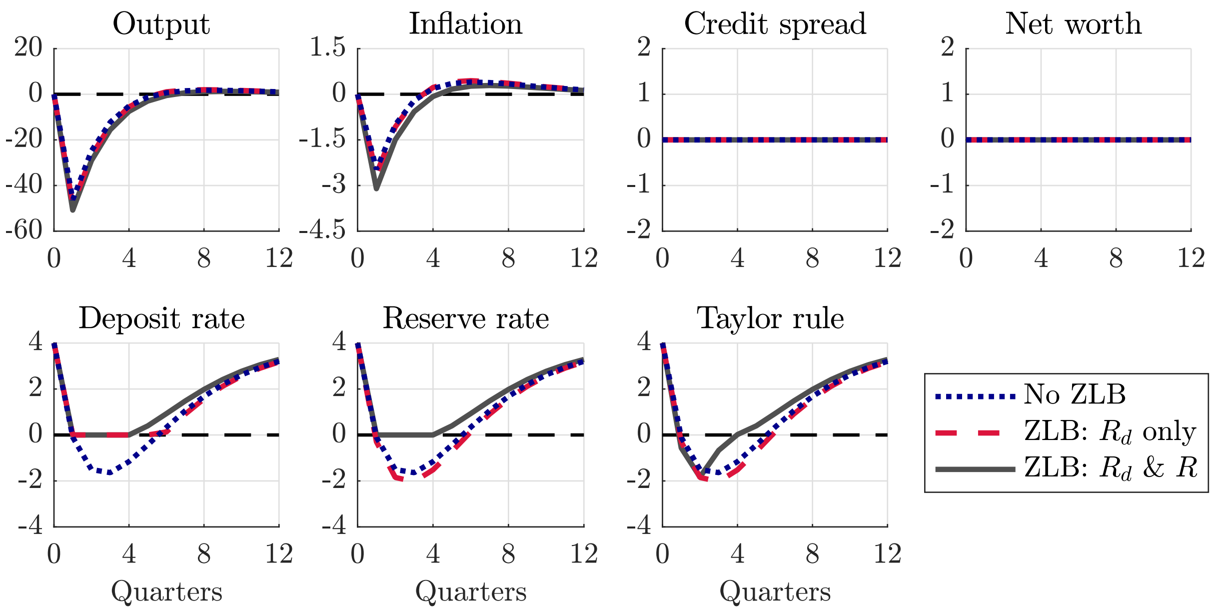
$$\begin{aligned} \phi_t = &-r_{d,t} + \theta \mathbb{E}_t \phi_{t+1} + \Phi\sigma (\mathbb{E}_t \phi_{t+1} - \phi_t + \mathbb{E}_t n_{t+1} - n_t) \\ &+ \Phi (\mathbb{E}_t \pi_{t+1} + s_t) - (\Phi - 1) \frac{r_{d,t} - \alpha r_t}{1-\alpha}. \end{aligned} \tag{A15}$$

Rearranging and setting $\theta = 0$ such that $n_t = 0$ gives Equation (15) in the main text. When $\theta > 0$, the model is described by five endogenous variables, $\{\pi_t, y_t, \phi_t, n_t, r_{d,t}\}$, and four private-sector conditions, (A9), (A12), (A13), and (A15).

A.3 Log-linear equilibrium: a Taylor-type rule [Section 2.2]

By replicating the key results from Section 3, this section shows that the stylized model captures the key features of the quantitative model. The experiments are conducted combining the IS and Phillips curve of the stylized model, Equations (13) and (16), respectively, and the Taylor-type rule of the quantitative model, (29).

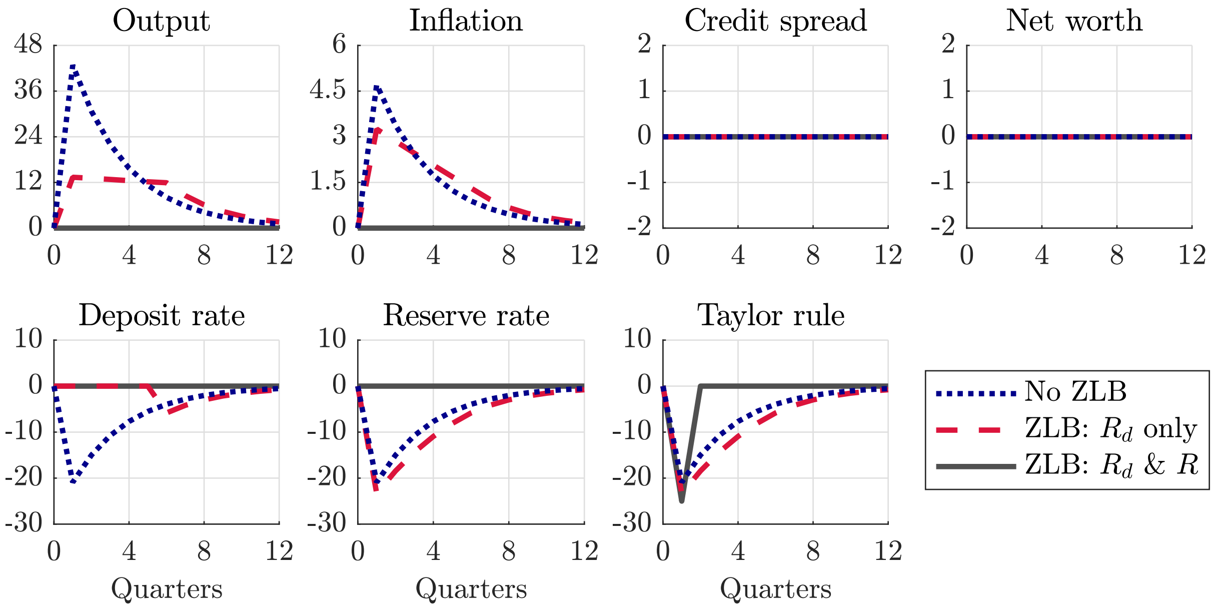
Figure A1: Stylized model: Natural rate shock with inertia in the policy rule



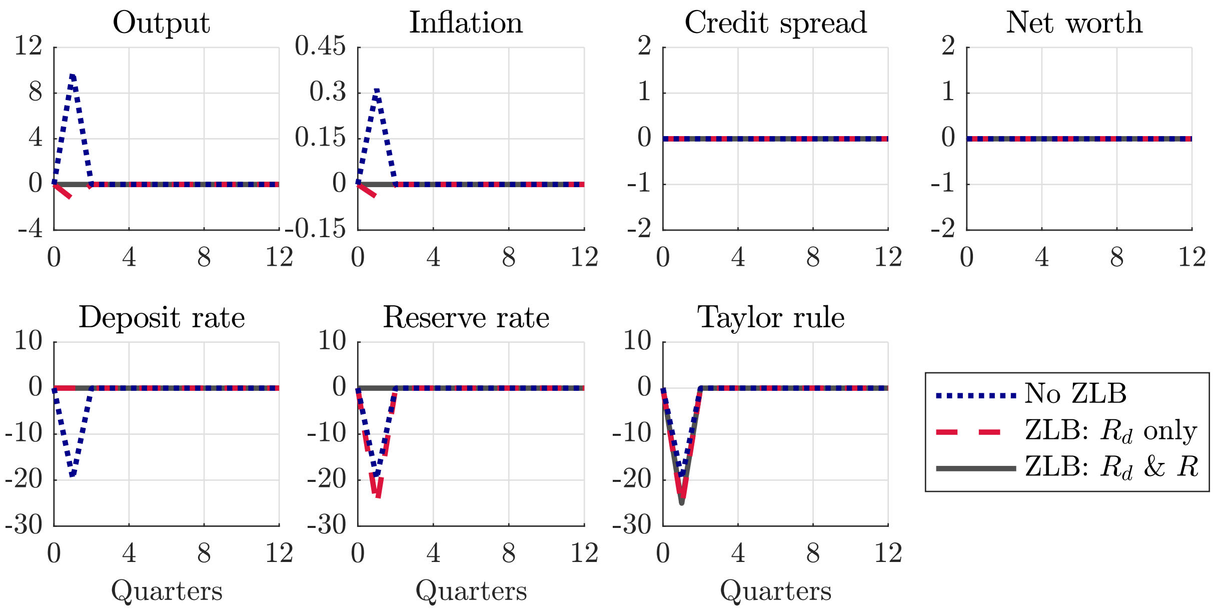
Note: Stylized model from Section 2 with a Taylor-type policy rule. $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a natural rate shock that brings the economy to the ZLB for 4 quarters. All interest rates displayed are in annualized percent. Other variables are in 100×log-deviation from steady state. Inflation is annualized.

Figure A2: Stylized model: Monetary policy shock in negative territory

(a) Policy rule with inertia

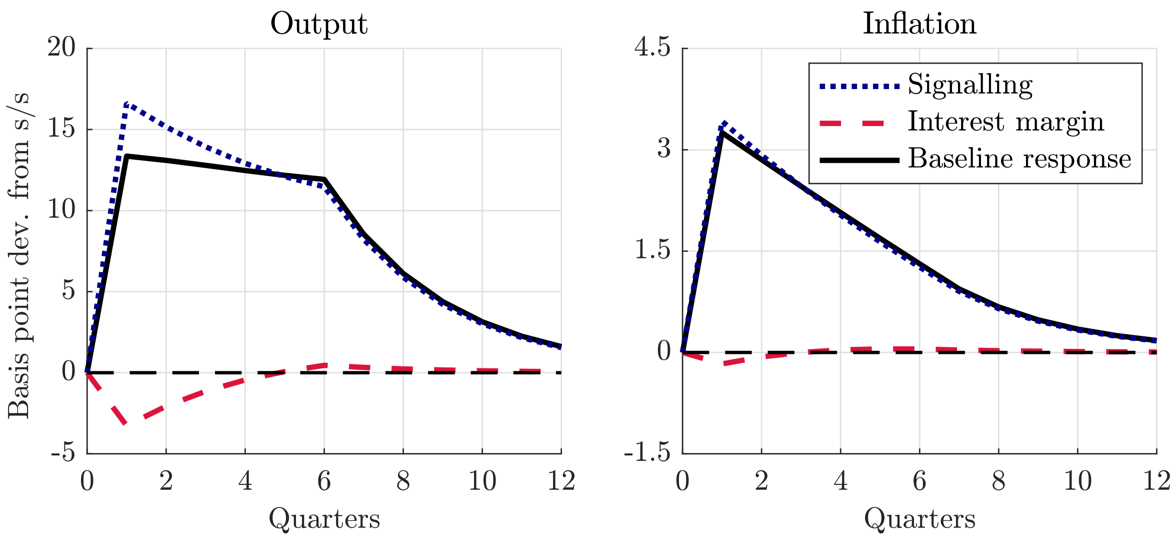


(b) Policy rule without inertia



Note: Stylized model from Section 2 with a Taylor-type policy rule. (a) $\alpha = 0.2$ and $\rho = 0.85$, (b) $\alpha = 0.2$ and $\rho = 0$. Impulse responses to a -25bp iid monetary policy shock at the ZLB. All interest rates displayed are in annualized basis points. Output and inflation are in basis point deviation from steady state. Inflation is annualized.

Figure A3: Stylized model: Contribution of signalling and interest margin channels



Note: Stylized model from Section 2 with a Taylor-type policy rule. Impulse responses to a −25bp iid monetary policy shock at the ZLB. Inflation is annualized. We linearly decompose the baseline response into "Signalling"—$\alpha = 0$ and $\rho = 0.85$, i.e. no costly interest margin channel—and "Interest margin"—difference between the baseline and "Signalling".

A.4 Analytical results: Propositions 2 and 3 [Section 2.3]

The recursive problem of the optimal policymaker is given by

$$V(\pi_{-1}, s) = \max_{\{\pi,y,r_d,r\}} -\frac{1}{2}\left(\pi^2 + \lambda y^2\right) + \beta \mathbb{E} V(\pi, s_{+1})$$

$$\pi = \beta \mathbb{E}\pi_{+1} + \gamma\pi_{-1} + \kappa y, \tag{PC}$$

$$y = \mathbb{E}y_{+1} - \sigma^{-1}\left(r_d - \mathbb{E}\pi_{+1} - s\right) - \phi\left(r_d - r\right), \tag{IS}$$

$$r_d \geq 0 \quad \text{(ZLB)}, \quad r_d - r \geq 0 \quad \text{(ARB)}, \quad r_d\left(r_d - r\right) = 0 \quad \text{(X)},$$

where the decentralized competitive equilibrium and a set of three inequality constraints on the policy tools constrain the optimal choice. This model is a slightly generalized version of the stylized model in Section 2. All proofs hold with lagged inflation added to the new-Keynesian Phillips Curve—resulting from, for example, price indexation.

Under **commitment**, the equilibrium can be summarized by the following equations:

$$\begin{aligned}
& & \pi &= \beta \mathbb{E}\pi_{+1} + \gamma\pi_{-1} + \kappa y, \\
& & y &= \mathbb{E}y_{+1} - \sigma^{-1}\left(r_d - \mathbb{E}\pi_{+1} - s\right) - \phi\left(r_d - r\right), \\
\pi: & & 0 &= \pi - \beta \mathbb{E}\boldsymbol{V_1}(\pi, s_{+1}) - \zeta_{PC} + \zeta_{PC_{-1}} + \sigma^{-1}\beta^{-1}\zeta_{IS_{-1}}, \\
y: & & 0 &= \lambda y + \kappa\zeta_{PC} - \zeta_{IS} + \beta^{-1}\zeta_{IS_{-1}}, \\
r_d: & & 0 &= \zeta_{IS}\left(\sigma^{-1} + \phi\right) + \zeta_{ZLB} + \zeta_{ARB} + \zeta_X\left(2r_d - r\right), \\
r: & & 0 &= \zeta_{IS}\phi + \zeta_{ARB} + \zeta_X r_d, \\
\text{KT}_1: & & 0 &= \zeta_{ZLB} r_d, \\
\text{KT}_2: & & 0 &= \zeta_{ARB}\left(r_d - r\right), \\
\text{EC}: & & \boldsymbol{V_1}(\pi_{-1}, s) &= -\gamma\zeta_{PC},
\end{aligned}$$

where the ζ are Lagrange multipliers. Based on the set of three inequality constraints on the policy tools, the following regimes can be defined: `Regime I`: $\{r_d > 0,\ r = r_d\}$, `Regime II`: $\{r_d = 0,\ r < 0\}$, and `Regime III`: $\{r_d = 0,\ r = 0\}$.

Proof of Proposition 2 Proposition 2 states that, under commitment, the reserve rate will never be set negative. This is equivalent to stating, $r \in$ `Regime II` is not optimal. We prove this by contradiction.

For a given state vector, $\boldsymbol{s} = \left\{\pi_{-1}, \zeta_{IS_{-1}}, \zeta_{PC_{-1}}, s\right\}$, define $r^{\text{c,zlb}}(\boldsymbol{s})$ and $r_d^{\text{c,zlb}}(\boldsymbol{s})$ as the reserve and deposit rate, respectively, that are the solution to the constrained commitment problem where negative rates are not allowed, $r \in \{$`Regime I, Regime III`$\}$, and $r^{\text{c,nir}}(\boldsymbol{s})$ and $r_d^{\text{c,nir}}(\boldsymbol{s})$ as the reserve and deposit rate that solve the commitment problem where negative reserve rates are allowed, i.e. $r \in \{$`Regime I, Regime II, Regime III`$\}$.

Consider $\phi > 0$. Suppose $\exists\ \boldsymbol{s} \mid V^{\text{c,nir}}(\boldsymbol{s}) > V^{\text{c,zlb}}(\boldsymbol{s}) \longrightarrow r^{\text{c,nir}} < 0$ and $r_d^{\text{c,nir}} = 0$ (`Regime II`). Then, the equilibrium allocation for $\{\pi, y\}$ is given by (PC) and (IS), where (IS) can be reduced to $y = \mathbb{E}y_{+1} + \sigma^{-1}(\mathbb{E}\pi_{+1} + s) + \phi r^{\text{c,nir}}$. Yet, $r^{\text{c,*}} = r_d^{\text{c,*}} = -\phi\sigma r^{\text{c,nir}} > 0$ (`Regime I`) generates the same equilibrium allocation, $V^{\text{c,*}}(\boldsymbol{s}) = V^{\text{c,nir}}(\boldsymbol{s})$. However, $r^{\text{c,*}}$ and $r_d^{\text{c,*}}$ are in the space of the constrained commitment problem such that $V^{\text{c,*}}(\boldsymbol{s}) = V^{\text{c,nir}}(\boldsymbol{s}) \leq V^{\text{c,zlb}}(\boldsymbol{s})$. Thus, we have a contradiction.

Consider $\phi = 0$. The reserve rate in this case drops out of the equilibrium system that determines $\{y, \pi, r_d, \zeta_{IS}, \zeta_{PC}\}$ as $\phi(r_d - r) = 0\ \forall\ r$ in (IS). There is no role for negative interest rates. ■

To study optimal time-consistent policy with and without policy smoothing, we augment the policymaker's objective function by adding a preference for smoothing interest rates, given by ψ. This gives the following, slightly modified, recursive planner's problem:

$$
\begin{aligned}
V(r_{-1}, \pi_{-1}, s) = \max_{\{\pi, y, r_d, r\}} & -\frac{1}{2}\left((1-\psi)\left(\pi^2 + \lambda y^2\right) + \psi\left(r - r_{-1}\right)^2\right) + \beta \mathbb{E} V(r, \pi, s_{+1}) \\
\pi &= \beta \mathbb{E}\pi_{+1} + \gamma \pi_{-1} + \kappa y, && \text{(PC)} \\
y &= \mathbb{E} y_{+1} - \sigma^{-1}\left(r_d - \mathbb{E}\pi_{+1} - s\right) - \phi\left(r_d - r\right), && \text{(IS)} \\
r_d &\geq 0 \quad \text{(ZLB)}, \quad r_d - r \geq 0 \quad \text{(ARB)}, \quad r_d\left(r_d - r\right) = 0 \quad \text{(X)}.
\end{aligned}
$$

Under **discretion**, the equilibrium can be summarized by the following equations:

$$
\begin{aligned}
& & \pi &= \beta \mathbb{E}\boldsymbol{\pi}\left(r, \pi, s_{+1}\right) + \gamma \pi_{-1} + \kappa y, \\
& & y &= \mathbb{E}\boldsymbol{y}\left(r, \pi, s_{+1}\right) - \sigma^{-1}\left(r_d - \mathbb{E}\boldsymbol{\pi}\left(r, \pi, s_{+1}\right) - s\right) - \phi\left(r_d - r\right), \\
\pi: & & 0 &= (1-\psi)\pi - \mathbb{E}\boldsymbol{V_2}\left(r, \pi, s_{+1}\right) - \zeta_{PC}\left(1 - \beta \mathbb{E}\boldsymbol{\pi_2}\left(r, \pi, s_{+1}\right)\right) \\
& & &\quad + \zeta_{IS}\left(\mathbb{E}\boldsymbol{y_2}\left(r, \pi, s_{+1}\right) + \sigma^{-1}\mathbb{E}\boldsymbol{\pi_2}\left(r, \pi, s_{+1}\right)\right), \\
y: & & 0 &= (1-\psi)\lambda y - \zeta_{IS} + \kappa \zeta_{PC}, \\
r_d: & & 0 &= \zeta_{IS}\left(\sigma^{-1} + \phi\right) + \zeta_{ZLB} + \zeta_{ARB} + \zeta_X\left(2r_d - r\right), \\
r: & & 0 &= \psi\left(r - r_{-1}\right) - \beta \mathbb{E}\boldsymbol{V_1}\left(r, \pi, s_{+1}\right) + \beta \mathbb{E}\boldsymbol{\pi_1}\left(r, \pi, s_{+1}\right)\zeta_{PC} \\
& & &\quad + \zeta_{IS}\left(\mathbb{E}\boldsymbol{y_1}\left(r, \pi, s_{+1}\right) + \sigma^{-1}\mathbb{E}\boldsymbol{\pi_1}\left(r, \pi, s_{+1}\right)\right) + \zeta_{ARB} + \zeta_X r_d, \\
\text{KT}_1: & & 0 &= \zeta_{ZLB} r_d, \\
\text{KT}_2: & & 0 &= \zeta_{ARB}\left(r_d - r\right), \\
\text{EC}_1: & & \boldsymbol{V}_1\left(r_{-1}, \pi_{-1}, S\right) &= -\psi\left(r - r_{-1}\right), \\
\text{EC}_2: & & \boldsymbol{V}_2\left(r_{-1}, \pi_{-1}, S\right) &= -\zeta_{PC}\gamma,
\end{aligned}
$$

where the ζ are Lagrange multipliers. Analogous to the commitment problem, once again the following three policy regimes can be defined: `Regime I`: $\{r_d > 0,\ r = r_d\}$, `Regime II`: $\{r_d = 0,\ r < 0\}$, and `Regime III`: $\{r_d = 0,\ r = 0\}$.

Proof of Proposition 3 Proposition 3 states that, under discretion, with $\psi = 0$, the reserve rate will never be set negative. Equivalently, $r \in$ `Regime II` is not optimal. We prove this by contradiction.

For a given state vector, $\boldsymbol{s} = \left\{r_{-1}, \pi_{-1}, \zeta_{IS_{-1}}, \zeta_{PC_{-1}}, s\right\}$, define $r^{\mathrm{d,zlb}}\left(\boldsymbol{s}\right)$ and $r_d^{\mathrm{d,zlb}}\left(\boldsymbol{s}\right)$ as the reserve and deposit rate, respectively, that are the solution to the constrained discretion problem where negative rates are not an option, $r \in \{$`Regime I, Regime III`$\}$, and $r^{\mathrm{d,nir}}\left(\boldsymbol{s}\right)$ and $r_d^{\mathrm{d,nir}}\left(\boldsymbol{s}\right)$ as the reserve and deposit rate that solve the discretion problem where negative reserve rates are allowed, i.e. $r \in \{$`Regime I, Regime II, Regime III`$\}$.

With $\psi = 0$, $\boldsymbol{V_1}\left(r_{-1}, \pi_{-1}, s\right) = 0$ and r_{-1} drops out as a state variable, i.e. expectations and allocations in the discretionary equilibrium are independent of r_{-1}. Thus, redefining $\boldsymbol{s} = \left\{\pi_{-1}, \zeta_{IS_{-1}}, \zeta_{PC_{-1}}, s\right\}$ we proceed as in the commitment case.

Consider $\phi > 0$: Suppose $\exists\ \boldsymbol{s} \mid V^{\mathrm{d,nir}}\left(\boldsymbol{s}\right) > V^{\mathrm{d,zlb}}\left(\boldsymbol{s}\right) \longrightarrow r^{\mathrm{d,nir}} < 0$ and $r_d^{\mathrm{d,nir}} = 0$ (`Regime II`). Then, the equilibrium allocation for $\{\pi, y\}$ is given by (PC) and (IS), where (IS) can be reduced to $y = \mathbb{E}\boldsymbol{y}\left(\pi, s_{+1}\right) + \sigma^{-1}\left(\mathbb{E}\boldsymbol{\pi}\left(\pi, s_{+1}\right) + s\right) + \phi r^{\mathrm{d,nir}}$. Yet, $r^{\mathrm{d,*}} = r_d^{\mathrm{d,*}} = -\phi\sigma r^{\mathrm{d,nir}} > 0$ (`Regime I`) generates the same equilibrium allocation, $V^{\mathrm{d,*}}\left(\boldsymbol{s}\right) = V^{\mathrm{d,nir}}\left(\boldsymbol{s}\right)$. However, $r^{\mathrm{d,*}}$ and $r_d^{\mathrm{d,*}}$ are in the space of the constrained commitment problem such that $V^{\mathrm{d,*}}\left(\boldsymbol{s}\right) = V^{\mathrm{d,nir}}\left(\boldsymbol{s}\right) \leq V^{\mathrm{d,zlb}}\left(\boldsymbol{s}\right)$. Thus, we have a contradiction.

Consider $\phi = 0$: The reserve rate in this case drops out of the equilibrium system that determines $\{y, \pi, r_d, \zeta_{IS}, \zeta_{PC}\}$ as $\phi\left(r_d - r\right) = 0\ \forall\ r$ in (IS). There is no role for negative interest rates. ■

A.5 Analytical results: additional state variables [Section 2.3]

This section discusses the possibility whether either r_{t-1} or alternative private-sector state variables (e.g., π_{t-1} and y_{t-1}) appearing in the private sector equilibrium conditions can generate results akin to our signalling channel, thus removing the need to assume a policy smoothing motive. The discussion proceeds in two parts.

Part I Suppose that the IS curve and an inertial monetary policy rule are given by

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t \pi_{t+1} - s_t\right), \tag{A16}$$

$$r_t = f\left(\pi_t, y_t\right) + \rho r_{t-1}. \tag{A17}$$

This policy rule can be written as a geometric distributed lag of past inflation and output,

$$\begin{aligned}
(1-\rho L)\, r_t &= f\left(\pi_t, y_t\right), \\
r_t &= \frac{\phi_\pi}{(1-\rho L)} f\left(\pi_t, y_t\right), \\
&= f\left(\pi_t, y_t\right) + \rho f\left(\pi_{t-1}, y_{t-1}\right) + \rho^2 f\left(\pi_{t-2}, y_{t-2}\right) + \cdots .
\end{aligned} \tag{A18}$$

By substitution, in an unconstrained environment, the equilibrium paths of output and inflation in (A16) and (A17) must be equivalent to the following equilibrium that features an IS curve with a lag structure in inflation and output; and a policy rule without inertia,

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t \pi_{t+1} - s_t + \rho f\left(\pi_{t-1}, y_{t-1}\right) + \rho^2 f\left(\pi_{t-2}, y_{t-2}\right) + \cdots\right), \tag{A19}$$

$$r_t = f\left(\pi_t, y_t\right). \tag{A20}$$

Alternatively, the IS curve can also be written in terms of lagged policy rates as follows

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(r_t - \mathbb{E}_t \pi_{t+1} - s_t + \rho r_{t-1} + \rho^2 r_{t-2} + \cdots\right). \tag{A21}$$

Thus, unconstrained by the ZLB, a model with an appropriately chosen structure of state variables in the private sector equilibrium conditions can replicate the equilibrium of the model in which the only state variable comes from Taylor rule inertia. We do not attempt to provide a micro-foundation for such a structure. However, an important insight is that a single lagged inflation or output term is not sufficient to replicate the equilibrium of the model with smoothing, especially when ρ is large (e.g., in the order of 0.8).

While inflation and output follow the same equilibrium path, the equilibrium interest rate path is quite different. In particular, the effect on impact ($t = 0$) for r_t in response to an exogenous shock is of the same magnitude. However, after that the equilibrium path of the interest rate without smoothing features faster mean reversion. This implies that once the ZLB constraint is introduced, the equilibrium paths for inflation and output are no longer equivalent across models since the numbers of periods spent at ZLB are different.

Part II More generally, we can write our 3-equation new-Keynesian model—abstracting from the costly interest margin channel—as

$$0 = \mathbb{E}_t f\left(\pi_{t+1}, y_{t+1}, \pi_t, y_t, r_t^d\right) = 0, \tag{A22}$$

$$r_t = g\left(\pi_t, y_t\right) + \varepsilon_t, \tag{A23}$$

$$r_t^d = \begin{cases} r_t & \text{if} \quad r_t \geq 0 \\ 0 & \text{if} \quad r_t < 0 \end{cases}, \tag{A24}$$

where (A22) incorporates the private-sector equations of the Phillips and IS curve. To see that adding endogenous state variables $\left(\pi_{t-1}, y_{t-1}, r_{t-1}^d\right)$ to the private-sector equilibrium (whether because of inflation indexation, consumption habits, or long-term bonds, respectively) does not generate an effective signalling channel of negative interest rates, consider the following experiment: Suppose $r_0^* = 0$ and the equilibrium path is defined by $\left\{\pi_t^*, y_t^*, r_t^{d*}, r_t^*\right\}_{t=0}^{\infty}$. In this case, a monetary policy shock, $\varepsilon_0 < 0$, lowers r_0 but since r_0^d remains unchanged at 0, this leaves the rest of the equilibrium path unchanged, irrespective of the presence of additional state variables in the private-sector equations.

This ineffectiveness of negative interest rates disappears if the Taylor rule contains a smoothing term, for example, as follows: $r_t = g\left(\pi_t, y_t\right) + \rho r_{t-1} + \varepsilon_t$. In this case, suppose $r_0 = 0$ and $r_1 > 0$. The same iid monetary policy shock in period 0 leaves r_0^d unchanged. However, all else equal, this shock lowers r_1 and hence r_1^d. Since r_1^d enters the private-sector equilibrium conditions, this alters the equilibrium path $\left\{\pi_t, y_t, r_t^d, r_t\right\}_{t=0}^{\infty}$.

Finally, note that the reserve rate, r_t does not enter (A22). It is difficult to conjecture a microfoundation (e.g., because of sticky information) in which the reserve rate would enter as a state variable, r_{t-1}, without the original presence of r_t. Of course, when $\phi > 0$, r_t does enter via the costly interest margin channel term in the IS curve, $-\phi\left(r_{d,t} - r_t\right)$, but this has an unambiguously negative sign on output. This is also the case in the quantitative model in Section 3 where the presence of r_{t-1} has an unambiguous negative sign in the banks' net worth accumulation equation.

A.6 Numerical results: policy function iteration [Section 2.4]

To derive a solution to the time-consistent optimal policymaker's problem, we use a policy function iteration algorithm, solving for $\pi(r,g)$, $y(r,g)$, $r'(r,g)$, $r_d(r,g)$, $\zeta_{ZLB}(r,g)$, and $\zeta_{ARB}(r,g)$. The algorithm proceeds as follows:

1. Set N_i: number of points on the interest rate grid, N_s: number of exogenous states, ϵ: tolerance limit for convergence, u: updating parameter. Set grid points $\{i_0,\ldots,i_{N_i}\}$. The AR(1) process for the natural rate, g, is approximated using Tauchen and Hussey (1991)'s quadrature algorithm that gives a set of grid points $\{s_0,\ldots,s_{N_s}\}$ and a transmission matrix, M.

2. Start iteration j with conjectured functions for $r'^j(r,g)$ and $\pi^j(r,g)$. The initial functions are set to $r'^0(r,g) = 1/\beta - 1$ and $\pi^0(r,g) = 0$. $\pi(r,g)$ is only defined at the nodes of the grids for the policy rate and shock, but since $r'(r,g)$ is generally not going to match node grids exactly, the function $\pi(r,g)$ is interpolated over the first argument to determine its values at $\boldsymbol{\pi}^j(r'^j(r,g),g')$. Construct expectations $\mathbb{E}\boldsymbol{\pi}^j(r'^j(r,g),g')$, denoted $\mathbb{E}\boldsymbol{\pi}^j$ for short. Repeat for r', giving $\mathbb{E}\boldsymbol{r}^j$.

3. Using the Phillips curve, calculate y:

$$y^j(r,g) = \frac{1}{\kappa}\left(\pi^j(r,g) - \mathbb{E}^j\boldsymbol{\pi}\right).$$

4. Construct one-step ahead output gap expectations, $\mathbb{E}\boldsymbol{y}^j$.

5. Construct the deposit rate function $r_d(r,g) = \max(0, r'^j(r,g))$.

6. Using the IS and Phillips curve, re-calculate y and π, respectively:

$$y^*(r,g) = \mathbb{E}\boldsymbol{y}^j - \sigma^{-1}\left(r_d(r,g) - \mathbb{E}\boldsymbol{\pi}^j - g\right) - \phi\left(r_d(r,g) - r'^j(r,g)\right),$$
$$\pi^*(r,g) = \beta\mathbb{E}\boldsymbol{\pi}^j + \kappa y^*(r,g),$$

   and then update expectations, $\mathbb{E}\boldsymbol{y}^*$ and $\mathbb{E}\boldsymbol{\pi}^*$.

7. Construct numerical derivatives of π as follows:

$$\pi_1(r,g) \equiv \frac{\partial \pi^*(r,g)}{\partial r} = \begin{cases} \frac{\pi^*(i_k,g)-\pi^*(i_{k-1},g)}{i_k - i_{k-1}} & \text{for} \quad k = 1,...,N_i, \\ \frac{\pi^*(i_1,g)-\pi^*(i_0,g)}{i_1 - i_0} & \text{for} \quad k = 0. \end{cases}$$

   and denote the function π_1 for short. Calculate the one-step ahead values of these derivative functions, $\boldsymbol{\pi}_1(r'^j(r,g),g')$, and calculate expectations, denoted $\mathbb{E}\boldsymbol{\pi}_1$. Repeat for y giving $\mathbb{E}\boldsymbol{y}_1$.

8. Using the FOC equation to re-calculate r':

`for` $r'^{j}(r,g) > 0,$

$$r'^{*}(r,g) = \frac{1}{\psi(1+\beta)}\begin{pmatrix} \psi r + \psi\beta\mathbb{E}\boldsymbol{r}^{j} - (1-\psi)\beta\mathbb{E}\boldsymbol{\pi}_1\pi^{*}(r,g) + \zeta^{*}_{ZLB}(r,g) \\ -(1-\psi)\left(\mathbb{E}\boldsymbol{y}_1 + \sigma^{-1}\mathbb{E}\boldsymbol{\pi}_1 - \sigma^{-1}\right)\left(\lambda y^{*}(r,g) + \kappa\pi^{*}(r,g)\right) \end{pmatrix}$$

`else`

$$r'^{*}(r,g) = \frac{1}{\psi(1+\beta)}\begin{pmatrix} \psi r + \psi\beta\mathbb{E}\boldsymbol{r}^{j} - (1-\psi)\beta\mathbb{E}\boldsymbol{\pi}_1\pi^{*}(r,g) + \zeta^{*}_{ZLB}(r,g) \\ -(1-\psi)\left(\mathbb{E}\boldsymbol{y}_1 + \sigma^{-1}\mathbb{E}\boldsymbol{\pi}_1 + \phi\right)\left(\lambda y^{*}(r,g) + \kappa\pi^{*}(r,g)\right) \end{pmatrix}$$

9. `if` $\max\left(\left(\pi^{*}(r,g) - \pi^{j}(r,g)\right), \left(r' * (r,g) - r'^{j}(r,g)\right)\right) < \epsilon$, then `stop`.
`else` $j = j+1$ and update the guess as follows:

$$\pi^{j}(r,g) = \mathtt{u}\pi^{j-1}(r,g) + (1-\mathtt{u})\,\pi^{*}(r,g)\,,$$
$$r'^{j}(r,g) = \mathtt{u}r'^{j-1}(r,g) + (1-\mathtt{u})\,r'^{*}(r,g)\,.$$

Repeat steps 2-9.

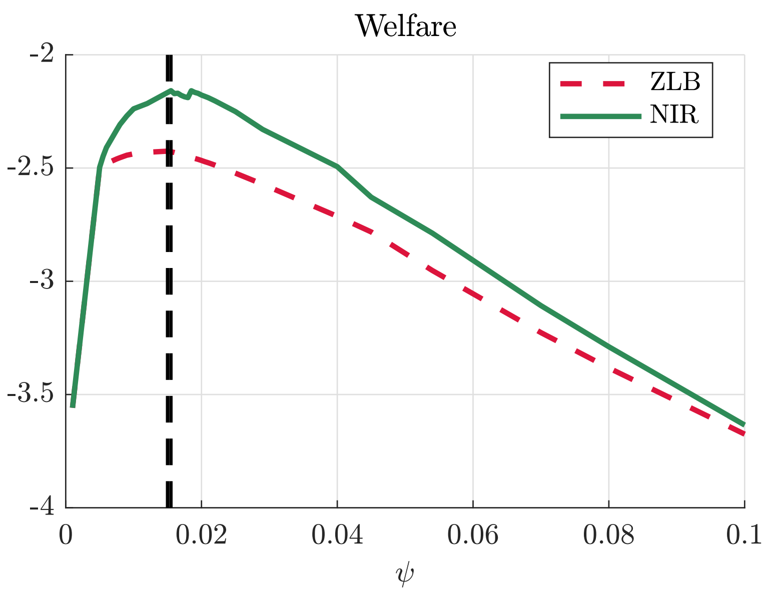
A.7 Numerical results: welfare computation [Section 2.4]

The social welfare function can be translated into a consumption equivalent measure via

$$CE = 100 \times (1 - \beta)\, \lambda^{-1} \left(\sigma^{-1} + \eta\right) \mathbb{E}\left(V^{SW}\right), \tag{A25}$$

where η is the inverse labor supply elasticity, set to 0.47 in our calibration, and $\mathbb{E}\left(V^{SW}\right)$ is the unconditional mean of the social welfare function. CE is the percentage of steady state consumption that the representative household would forgo in each period to avoid uncertainty. Less negative values thus represent an improvement in welfare. Figure A4 plots the consumption equivalent measure of welfare across a range of values for the smoothing parameter, ψ. It demonstrates three features. One, allowing for negative interest rates in the toolkit of the policymaker is weakly welfare dominant. Two, it is optimal to delegate policy to a central banker with a small but meaningful preference for smoothing. Three, the optimal value of ψ is virtually the same, irrespective of whether negative interest rates are available or not.

Figure A4: Welfare and the optimal degree of smoothing

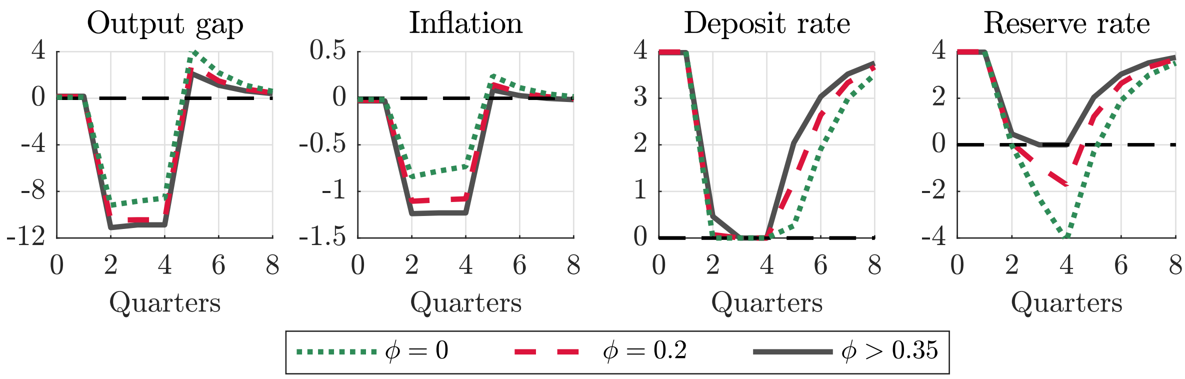


Note: Consumption equivalent in percent of steady state consumption. Black-dash is the optimal value of ψ. "ZLB" denotes policy without negative interest rates. "NIR" denotes policy with negative rates.

A.8 Numerical results: optimal policy experiment [Section 2.4]

In this section, we add one more set of results to our investigation regarding the optimality of negative rates. Figure A5 shows an experiment in which the natural real rate, s_t, drops into negative territory and remains at that level for 3 quarters before returning to steady state. The red-dash line is our baseline parameterization. The black-solid line is the equilibrium outcome when the policymaker is not able to set a negative reserve rate (or, equivalently, when the cost of negative interest rates is sufficiently high—in this case $\phi > 0.35$—such that the policymaker chooses not to use negative interest rates). The green-dotted line plots an extreme scenario where there is no cost of negative interest rates ($\phi = 0$).

Figure A5: Optimal policy scenarios



Note: Impulse responses to a drop in s_t into negative territory for 3 quarters before jumping back to its steady state value. The output gap is measured in percent. Inflation is in annualized percent deviation from steady state. The deposit and reserve rates are in levels, annualized.

When $\phi > 0.35$, the policymaker behaves as if there was a ZLB on the reserve rate. The nominal reserve rate is lowered to the ZLB, but this easing does not generate a sufficient fall in the real deposit rate, $r_{d,t} - \mathbb{E}_t\pi_{t+1}$, to offset the fall in s_t. As a result, inflation falls and the output gap opens. In contrast, when $\phi = 0.2$ the policymaker gradually lowers the reserve rate into negative territory, reaching -1.2% in period 4. Although the deposit rate remains bounded by zero, this negative reserve rate ensures that the deposit rate is lower after period 4 than without negative interest rates. This lower path for the deposit rate allows inflation to overshoot after s_t is back at steady state, also lowering the expected real deposit rate in early periods. As a consequence the drop in inflation and the widening of the output gap is less severe. The scenario without the cost of negative rates ($\phi = 0$) shows the maximum impact of negative interest rates. In this case, the reserve rate reaches -3.8% in period 2 and the deposit rate is a full 1 percentage point lower in period 6 than in the case without negative rates. The drop in the output gap and inflation is much less pronounced than in the other two scenarios.

This exercise illustrates that the increased frequency at the ZLB arises for two reasons: First, signalling with negative rates keeps the deposit rate lower-for-longer in response to a contractionary shock. Second, on impact the policymaker with access to negative rates is willing to cut the policy rate faster. Observe that, due to smoothing, the black-solid line does not reach the ZLB until period 3 as the benefit of cutting the period-2 policy rate further is outweighed by the cost in terms of smoothing rates. In contrast, the red-dash and green-dot lines (negative rate scenarios) already reach the ZLB in period 2.

A.9 Comparative statics: closed-form solutions [Section 2.4]

In Section 2.4 we set $\lambda = 0$, we set $\psi = 0$ except for between periods 1 and 2, and $s_t = 0$ for $t > 1$. This allows for an analytical derivation of equilibrium outcomes. In particular, $\pi_t = y_t = 0$ for $t > 2$. Thus, the central banks loss function reduces to

$$-V \propto \pi_1^2 + \beta\left((1-\psi)\,\pi_2^2 + \psi\,(r_2 - r_1)^2\right). \tag{A26}$$

The policymaker is subject to the following constraints

$$\pi_1 = \beta\pi_2 + \kappa y_1, \tag{A27}$$

$$y_1 = y_2 - \sigma^{-1}\left(r_{d,1} - \pi_2\right) - \phi\left(r_{d,1} - r_1\right) + g, \tag{A28}$$

$$\pi_2 = \kappa y_2, \tag{A29}$$

$$y_2 = -\sigma^{-1} r_2, \tag{A30}$$

$$r_{d,1} \geq -\bar{r}, \tag{A31}$$

$$r_2 \geq -\bar{r}, \tag{A32}$$

$$r_{d,1} - r_1 \geq 0, \tag{A33}$$

$$\left(r_{d,1} + \bar{r}\right)\left(r_{d,1} - r_1\right) = 0, \tag{A34}$$

where the expectations operator has been dropped because there is no uncertainty. In addition, there is no incentive to set a negative interest rate in period 2 so $r_{d,2} = r_2$. In contrast to the main text, we make g mean zero and set the ZLB constraint as $-\bar{r}$.

We consider optimal policy under discretion. There are 4 possible equilibrium outcomes:

$$(++):\quad r_1 > -\bar{r}, \quad r_2 > -\bar{r}, \tag{A35}$$

$$(0+):\quad r_1 = -\bar{r}, \quad r_2 > -\bar{r}, \tag{A36}$$

$$(-+):\quad r_1 < -\bar{r}, \quad r_2 > -\bar{r}, \tag{A37}$$

$$(-0):\quad r_1 < -\bar{r}, \quad r_2 = -\bar{r}. \tag{A38}$$

We solve the problem backwards. First solving for the optimal r_2 given a value for r_1.

For $(\cdot 0)$, we have

$$r_2^{*(0)} = -\bar{r}. \tag{A39}$$

For $(\cdot +)$, the period 2 problem is given by

$$\min_{r_2} \quad (1-\psi)\,\pi_2^2 + \psi\,(r_2 - r_1)^2 \quad \text{s.t.} \quad \pi_2 = -\kappa\sigma^{-1} r_2. \tag{A40}$$

The first-order condition is given by

$$(1-\psi)\left(\kappa\sigma^{-1}\right)^2 r_2 + \psi\left(r_2 - r_1\right) = 0, \tag{A41}$$

or, rearranged, as

$$r_2^{*(+)} = R_2^{(+)} r_1, \tag{A42}$$
$$\pi_2^{*(+)} = \Pi_2^{(+)} r_1, \tag{A43}$$
$$\text{where} \quad R_2^{(+)} \equiv \frac{\psi}{\psi + (1-\psi)\left(\kappa\sigma^{-1}\right)^2}, \tag{A44}$$
$$\Pi_2^{(+)} \equiv -\kappa\sigma^{-1} R_2^{(+)}. \tag{A45}$$

Now that we have the optimal reaction function for r_2 as a function of r_1, we can solve the period 1 problem, taking the behaviour of the policymaker in period 2 as given.

For (++), the period 1 problem is given by

$$\min_{r_1} \quad \pi_1^2 + \beta\left((1-\psi)\,\pi_2^2 + \psi\left(r_2 - r_1\right)^2\right) \tag{A46}$$
$$\text{s.t.} \quad \pi_1 = \Pi_1^{(++)} r_1 + \kappa g, \tag{A47}$$
$$\pi_2 = \Pi_2^{(+)} r_1, \tag{A48}$$
$$r_2 = R_2^{(+)} r_1, \tag{A49}$$
$$\text{where} \quad \Pi_1^{(++)} \equiv -\kappa\left(\left(\beta + 1 + \kappa\sigma^{-1}\right)\sigma^{-1} R_2^{(+)} + \sigma^{-1}\right), \tag{A50}$$

and the first-order condition is given by

$$\left(\Pi_1^{(++)} r_1 + \kappa g\right)\Pi_1^{(++)} + \beta\left((1-\psi)\left(\Pi_2^{(+)}\right)^2 r_1 + \psi\left(R_2^{(+)} - 1\right)^2 r_1\right) = 0, \tag{A51}$$

or, rearranged, as

$$r_1^{*(++)} = -\frac{\kappa\Pi_1^{(++)} g}{\left(\Pi_1^{(++)}\right)^2 + \beta\left((1-\psi)\left(\Pi_2^{(+)}\right)^2 + \psi\left(R_2^{(+)} - 1\right)^2\right)}. \tag{A52}$$

For $(-+)$, the constraints are given by

$$\pi_1 = \Pi_1^{(-+)} r_1 + C\Pi_1^{(-+)} \bar{r} + \kappa g, \tag{A53}$$

$$\pi_2 = \Pi_2^{(+)} r_1, \tag{A54}$$

$$r_2 = R_2^{(+)} r_1, \tag{A55}$$

$$\text{where} \quad \Pi_1^{(-+)} \equiv -\kappa \left(\left(\beta + 1 + \kappa\sigma^{-1} \right) \sigma^{-1} R_2^{(+)} - \phi \right), \tag{A56}$$

$$C\Pi_1^{(-+)} \equiv \kappa \left(\sigma^{-1} + \phi \right), \tag{A57}$$

and the solution is given by

$$r_1^{*(-+)} = -\frac{C\Pi_1^{(-+)} \Pi_1^{(-+)} \bar{r} + \kappa \Pi_1^{(-+)} g}{\left(\Pi_1^{(-+)}\right)^2 + \beta \left((1-\psi) \left(\Pi_2^{(+)}\right)^2 + \psi \left(R_2^{(+)} - 1 \right)^2 \right)}. \tag{A58}$$

For $(0+)$, we have

$$r_1^{*(0+)} = -\bar{r}. \tag{A59}$$

For (-0), the constraints are given by

$$\pi_1 = \Pi_1^{(-0)} r_1 + C\Pi_1^{(-0)} \bar{r} + \kappa g, \tag{A60}$$

$$\pi_2 = C\Pi_2^{(0)} \bar{r}, \tag{A61}$$

$$r_2 = -\bar{r}, \tag{A62}$$

$$\text{where} \quad \Pi_1^{(-0)} \equiv \kappa\phi, \tag{A63}$$

$$C\Pi_1^{(-0)} \equiv \kappa \left(\left(\beta + 2 + \kappa\sigma^{-1} \right) \sigma^{-1} + \phi \right), \tag{A64}$$

$$C\Pi_2^{(0)} \equiv \kappa\sigma^{-1}, \tag{A65}$$

and the first-order condition is given by

$$\left(\Pi_1^{(-0)} r_1 + C\Pi_1^{(-0)} \bar{r} + \kappa g \right) \Pi_1^{(-0)} + \beta\psi \left(\bar{r} + r_1 \right) = 0, \tag{A66}$$

or, rearranged, as

$$r^{*(-0)} = -\frac{\Pi_1^{(-0)} C\Pi_1^{(-0)} \bar{r} + \Pi_1^{(-0)} \kappa g + \beta\psi\bar{r}}{\left(\Pi_1^{(-0)}\right)^2 + \beta\psi}. \tag{A67}$$

This completes the full set of equilibrium conditions. Numerically, we solve for each possible case and throw out any solutions which violate the assumptions of that case. If multiple solutions exist, we choose the one that maximizes welfare.

Next, we use these analytical results to prove Propositions 5 and 6 in the main text.

Proof of Proposition 5 Proposition 5 states that there exists a threshold ϕ^* below which a negative interest rate policy is effective in both raising inflation π_1 and output y_1. We prove this as follows.

First, we assume the size of g ensures that $r_2 = r_{d,2} > 0$, $r_{d,1} = 0$, and $r < 0$. Second, r_2 is set optimally as in equation (A42). Third, we substitute into the period 1 Phillips curve in order to write π_1 in terms of r_1. This is given by

$$\pi_1 = -\kappa \left(\left(\beta + 1 + \kappa\sigma^{-1}\right) \sigma^{-1} \frac{\psi}{\psi + (1-\psi)\left(\kappa\sigma^{-1}\right)^2} - \phi \right) r_1. \tag{A68}$$

The condition for negative rates to be effective, $\partial\pi_1/\partial r_1 < 0$, therefore holds when

$$\phi < \phi_\pi^* = \left(\beta + 1 + \kappa\sigma^{-1}\right) \sigma^{-1} \frac{\psi}{\psi + (1-\psi)\left(\kappa\sigma^{-1}\right)^2}. \tag{A69}$$

Note the threshold for raising output in period 1, ϕ_y^*, is more demanding. In particular,

$$\phi_y^* = \left(1 + \kappa\sigma^{-1}\right) \sigma^{-1} \frac{\psi}{\psi + (1-\psi)\left(\kappa\sigma^{-1}\right)^2}. \tag{A70}$$

Hence, it is possible, if $\phi_y^* < \phi < \phi_\pi^*$, that negative rates raise inflation while causing a contraction in output. Setting $\phi^* = \phi_y^*$ completes the proof. ■

Proof of Proposition 6 Proposition 6 states that the cut in the policy rate needed to generate the same effect on output (and inflation) is larger in negative than in positive territory. We prove this as follows.

One, the IS equation in period 1 can be rewritten as a relationship between y_1 and g,

$$y_1(g) = y_2(r_1(g)) - \sigma^{-1}(r_{d,1}(r_1(g)) - \pi_2(r_1(g))) - \phi(r_{d,1}(r_1(g)) - r_1(g)) + g. \quad \text{(A71)}$$

Two, note that $\partial r_{d,1}/\partial r_1 = 1$ when $r_1 > 0$ and $\partial r_{d,1}/\partial r_1 = 0$ when $r_1 < 0$. Three, since we assume g is such that $r_2 > 0$, it follows that $\partial \pi_2/\partial r_1$ and $\partial y_2/\partial r_1$ are common across both $r_1 < 0$ and $r_1 > 0$ scenarios. Four, note that $\partial \pi_1/\partial r_1$ only differs across scenarios in so far as $\partial y_1/\partial r_1$ differs across scenarios. Hence, when evaluating the effectiveness of policy, we need only concern ourselves with $\partial y_1/\partial r_1$. Five, let us evaluate the response $\partial r_1/\partial g$ that ensures $dy_1/dg = 0$. When $r_1 < 0$ and $r_2 > 0$, the derivative $\frac{dy_1}{dg} = 0$ is

$$0 = \frac{\partial y_2}{\partial r_1}\frac{\partial r_1}{\partial g} - \sigma^{-1}\left(-\frac{\partial \pi_2}{\partial r_1}\frac{\partial r_1}{\partial g}\right) - \phi\left(-\frac{\partial r_1}{\partial g}\right) + 1, \quad \text{(A72)}$$

$$\implies \left.\frac{\partial r_1}{\partial g}\right|_{r_1<0,r_2>0} = \frac{1}{\left(-\frac{\partial y_2}{\partial r_1} - \sigma^{-1}\frac{\partial \pi_2}{\partial r_1} - \phi\right)}. \quad \text{(A73)}$$

In the scenario where $r_1, r_2 > 0$, the derivative is given by

$$0 = \frac{\partial y_2}{\partial r_1}\frac{\partial r_1}{\partial g} - \sigma^{-1}\left(\frac{\partial r_1}{\partial g} - \frac{\partial \pi_2}{\partial r_1}\frac{\partial r_1}{\partial g}\right) + 1, \quad \text{(A74)}$$

$$\implies \left.\frac{\partial r_1}{\partial g}\right|_{r_1,r_2>0} = \frac{1}{\left(-\frac{\partial y_2}{\partial r_1} - \sigma^{-1}\frac{\partial \pi_2}{\partial r_1} + \sigma^{-1}\right)}. \quad \text{(A75)}$$

Next, we assume that $-\left(\frac{\partial y_2}{\partial r_1} + \sigma^{-1}\frac{\partial \pi_2}{\partial r_1}\right) > \phi$. This is equivalent to the threshold condition in Proposition 5 that ensures negative rates are effective. If this condition holds and negative rates are effective, then the proof reduces to $\sigma^{-1} > -\phi$, which is always true.

Finally, note that the proof follows the same steps if started from $d\pi_1/dg = 0$. ■

B Quantitative model

Appendix B relates to Section 3 (main paper) on the effectiveness of negative rates in a quantitative new-Keynesian model. Section B.1 shows how to derive the financial sector equilibrium in just two equations. Section B.2 documents the complete set of equilibrium equations. Section B.3 provides further information on the parameterization of the model regarding calibration targets, estimation method and results, as well as data sources and treatment. Section B.4 reports additional empirical evidence on policy smoothing. Section B.5 shows that without interest rate inertia the signalling channel is not active. Section B.6 derives the bank profit decomposition in our baseline model and in an extended version of the model with firm equity and loan finance. Sections B.7 documents the robustness of our results with respect to changes in the Frisch labor supply elasticity, the Phillips curve slope, the investment elasticity, and the introduction of nominal wage rigidities. Finally, Section B.8 summarizes the necessary changes to the equilibrium system of equations when nominal wage rigidities are introduced.

B.1 Set up: derivation of the banker's problem [Section 3.1]

A banker j solves

$$V_{n,t}(j) = \max_{\{S_t(j),A_t(j),D_t(j),N_t(j)\}} \mathbb{E}_t \Lambda_{t,t+1} \left((1-\theta) N_{t+1}(j) + \theta V_{n,t+1}(j) \right), \tag{B1}$$

subject to

$$Q_t S_t(j) + A_t(j) = D_t(j) + N_t(j), \tag{B2}$$
$$V_{n,t}(j) \geq \lambda Q_t S_t(j), \tag{B3}$$
$$A_t(j) = \alpha(x_t) D_t(j), \tag{B4}$$
$$N_t(j) = R_{k,t} Q_{t-1} S_{t-1}(j) + \frac{R_{t-1}}{\Pi_t} A_{t-1}(j) - \frac{R_{d,t-1}}{\Pi_t} D_{t-1}(j),, \tag{B5}$$

where the constraints are the balance sheet constraint, incentive compatibility constraint, reserve ratio, and net worth accumulation, respectively. We calibrate the model such that the incentive constraint is always binding. Next, we simplify the system of constraints by substituting reserves, $A_t(j)$, and deposits, $D_t(j)$, making use of Equations (B2) and (B4). We also define $\Phi_t \equiv Q_t S_t(j)/N_t(j)$ to be the leverage ratio of a banker (and Φ_t is common across banks). Thus, the accumulation of net worth, (B5), is given by

$$N_t(j) = \left(R_{k,t} \Phi_{t-1} - \frac{R_{d,t-1} - \alpha(x_t) R_{t-1}}{(1-\alpha(x_t)) \Pi_t} (\Phi_{t-1} - 1) \right) N_{t-1}(j). \tag{B6}$$

Furthermore, we conjecture the value function to take the form

$$V_{n,t}(j) = (\zeta_{s,t} \Phi_t + \zeta_{n,t}) N_t(j), \tag{B7}$$

where $\zeta_{s,t}$ and $\zeta_{n,t}$ are as yet undetermined.

Substituting (B6) and (B7), the banker's problem can be rewritten as

$$\begin{aligned}(\zeta_{s,t}\Phi_t + \zeta_{n,t}) = \max_{\Phi_t} \mathbb{E}_t \Lambda_{t,t+1} &\left((1-\theta) + \theta (\zeta_{s,t+1} \Phi_{t+1} + \zeta_{n,t+1}) \right) \\ &\times \left(R_{k,t+1} \Phi_t - \frac{R_{d,t} - \alpha(x_{t+1}) R_t}{(1-\alpha(x_{t+1})) \Pi_{t+1}} (\Phi_t - 1) \right),\end{aligned} \tag{B8}$$

subject to

$$\zeta_{s,t} \Phi_t + \zeta_{n,t} = \lambda \Phi_t. \tag{B9}$$

We rearrange the incentive compatibility constraint (B9) and iterate one period forward to find optimal (and maximum) leverage given by

$$\Phi_{t+1} = \frac{\zeta_{n,t+1}}{\lambda - \zeta_{s,t+1}}. \tag{B10}$$

With (B10), comparing the left and right hand side of (B8), we verify the conjectured functional form of the value function. This allows us to summarize the solution to the financial intermediary's problem in the binding incentive constraint given by

$$\lambda\Phi_t = \mathbb{E}_t\Lambda_{t,t+1}\left((1-\theta)+\theta\lambda\Phi_{t+1}\right)\left(R_{k,t+1}\Phi_t - \frac{R_{d,t}-\alpha\left(x_{t+1}\right)R_t}{\left(1-\alpha\left(x_{t+1}\right)\right)\Pi_{t+1}}\left(\Phi_t - 1\right)\right). \tag{B11}$$

Aggregate net worth in the financial sector evolves as a weighted sum of existing banks' accumulated net worth (B6) and start up funds new banks receive from the household. Entering banks receive a fraction ω of the total value of intermediated assets, i.e. $\omega Q_t S_{t-1}$. In equilibrium, $S_t = K_t$. Thus, the evolution of aggregate net worth is given by

$$N_t = \theta\left(R_{k,t}\Phi_{t-1} - \frac{R_{d,t-1}-\alpha\left(x_t\right)R_{t-1}}{\left(1-\alpha\left(x_t\right)\right)\Pi_t}\left(\Phi_{t-1}-1\right)\right)N_{t-1} + \omega Q_t K_{t-1}. \tag{B12}$$

Equations (B11) and (B12) express the financial sector problem in just two equations. This completes the derivation.

B.2 Set up: list of equilibrium conditions [Section 3.1]

In equilibrium, we summarize the quantitative model in 23 equations in 23 endogenous variables, $\{Y_t$, $Y_{m,t}$, L_t, C_t, $\tilde{C}_t$, $\Lambda_{t,t+1}$, μ_t, K_t, I_t, $I_{n,t}$, N_t, Φ_t, Δ_t, W_t, Π_t, X_t, $P_{m,t}$, Q_t, $R_{k,t}$, $R_{T,t}$, R_t, $R_{d,t}$, $CS_t\}$, and 3 exogenous processes, $\{\zeta_t$, ϵ_t, $\varepsilon_{m,t}\}$. Government expenditure, G, is financed via lump-sum taxes and kept constant.

Households

- Euler equation

$$1 = \mathbb{E}_t \Lambda_{t,t+1} \exp(\zeta_t) R_{d,t}/\Pi_{t+1} \tag{B13}$$

- Labor supply

$$\mu_t W_t = \chi L_t^{\varphi} \tag{B14}$$

- Stochastic discount factor

$$\Lambda_{t,t+1} = \beta \mu_{t+1}/\mu_t \tag{B15}$$

- Marginal utility of consumption

$$\mu_t = \tilde{C}_t^{-\sigma} - \beta \hbar \mathbb{E}_t \tilde{C}_{t+1}^{-\sigma} \tag{B16}$$

Financial intermediaries

- Incentive compatibility constraint

$$\lambda \Phi_t = \mathbb{E}_t \Lambda_{t,t+1} ((1-\theta) + \theta \lambda \Phi_{t+1}) \left(R_{k,t+1} \Phi_t - \frac{R_{d,t} - \alpha(x_t) R_t}{(1-\alpha(x_t)) \pi_{t+1}} (\phi_t - 1) \right) \tag{B17}$$

- Evolution of aggregate net worth

$$N_t = \theta \left(R_{k,t} \Phi_{t-1} - \frac{R_{d,t-1} - \alpha(x_t) R_{t-1}}{(1-\alpha(x_t)) \Pi_t} (\Phi_{t-1} - 1) \right) N_{t-1} + \omega Q_t K_{t-1} \tag{B18}$$

Intermediate goods firms

- Price of capital

$$\begin{aligned} 1 = Q_t &\left(1 - \frac{\eta}{2} \left(\frac{I_{n,t} - I_{n,t-1}}{I_{n,t-1} + I} \right)^2 - \eta \frac{I_{n,t} - I_{n,t-1}}{(I_{n,t-1} + I)^2} I_{n,t} \right) \\ &+ \mathbb{E}_t \Lambda_{t,t+1} Q_{t+1} \left(\eta (I_{n,t+1} - I_{n,t}) \frac{I_{n,t+1} + I}{(I_{n,t} + I)^3} I_{n,t+1} \right) \end{aligned} \tag{B19}$$

- Production function

$$Y_{m,t} = K_{t-1}^{\gamma} L_t^{1-\gamma} \tag{B20}$$

- Labor demand

$$W_t = (1-\gamma)\, P_{m,t} Y_{m,t} / L_t \tag{B21}$$

- Return on capital

$$R_{k,t} = \frac{\gamma P_{m,t} Y_{m,t} / K_{t-1} + Q_t - \delta}{Q_{t-1}} \tag{B22}$$

Retail firms

- Price Phillips curve

$$\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\mathcal{D}_t}{\mathcal{F}_t} = \left(\frac{1-\iota\,\Pi_t^{\epsilon-1}}{1-\iota}\right)^{\frac{1}{1-\epsilon}}. \tag{B23}$$

$$\text{where}\quad \mathcal{D}_t \equiv \mu_t P_{m,t} Y_t + \beta\iota\mathbb{E}_t\,\Pi_{t+1}^{\epsilon}\mathcal{D}_{t+1},$$
$$\mathcal{F}_t \equiv \mu_t Y_t + \beta\iota\mathbb{E}_t\,\Pi_{t+1}^{\epsilon-1}\mathcal{F}_{t+1}.$$

- Price dispersion

$$\Delta_t = (1-\iota)\left[\left(\frac{\epsilon}{\epsilon-1}\right)\frac{\mathcal{D}_t}{\mathcal{F}_t}\right]^{-\epsilon} + \iota\Pi_t^{\epsilon}\Delta_{t-1} \tag{B24}$$

Monetary policy

- Policy rule

$$R_{T,t} = \left(R\Pi_t^{\phi_\pi}\left(\frac{X_t}{X}\right)^{\phi_x}\right)^{1-\rho} R_{t-1}^{\rho}\exp\left(\varepsilon_{m,t}\right) \tag{B25}$$

- No arbitrage

$$\begin{aligned} &\text{(I)} \quad R_t = R_{d,t} = R_{T,t}\ ,\ \text{or} \\ &\text{(II)} \quad R_t = R_{d,t} = \max\{1, R_{T,t}\}\ ,\ \text{or} \\ &\text{(III)} \quad R_t = R_{T,t} \quad \text{and} \quad R_{d,t} = \max\{1, R_{T,t}\}\,. \end{aligned} \tag{B26}$$

General equilibrium

- Aggregate output

$$Y_t = Y_{m,t} / \Delta_{p,t}, \tag{B28}$$

- Aggregate resource constraint

$$Y_t = C_t + I_t + G \tag{B29}$$

- Capital accumulation

$$K_t = K_{t-1} + f\left(I_{n,t}, I_{n,t-1}\right), \tag{B30}$$

$$\text{where} \quad f\left(I_{n,t}, I_{n.t-1}\right) \equiv \left(1 - (\eta/2)\left(\left(I_{n,t} + I_{n,t-1}\right)/\left(I_{n,t-1} + I\right)\right)^2\right) I_{n,t}.$$

Further definitions

- Habit adjusted consumption

$$\tilde{C}_t = C_t - \hbar C_{t-1} \tag{B31}$$

- Total investment

$$I_t = I_{n,t} + \delta K_{t-1} \tag{B32}$$

- Leverage

$$\Phi_t = Q_t K_t / N_t \tag{B33}$$

- Marginal cost

$$X_t = P_{m,t} \tag{B34}$$

- Credit spread

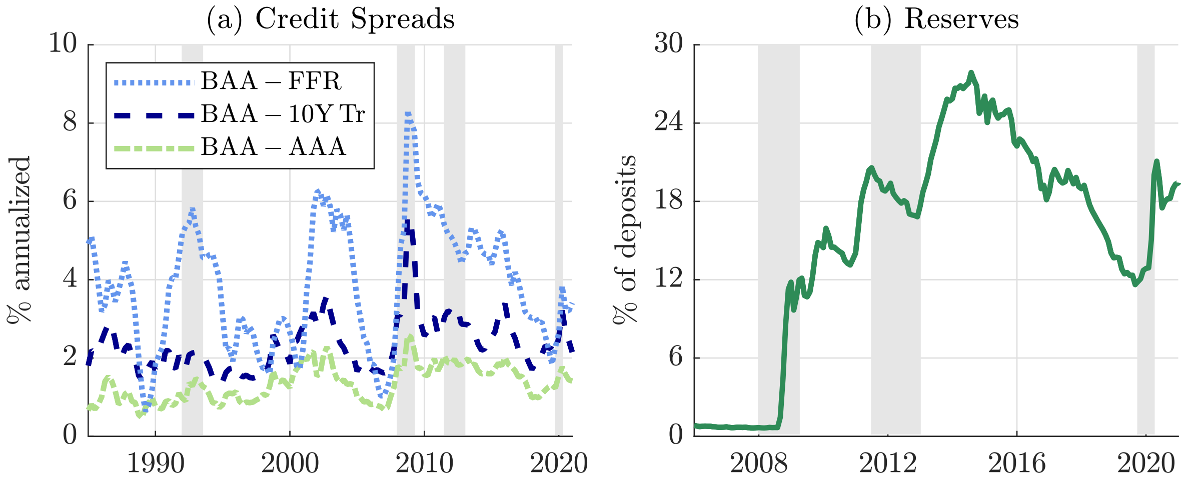
$$CS_t = R_{k,t+1} / \left(R_{d,t}/\Pi_{t+1}\right) \tag{B35}$$

B.3 Parameterization: further details [Section 3.2]

Table 1 in the main text presents the baseline parameterization of the quantitative model. The parameters are grouped into three blocks. Block A contains structural parameters that are assigned standard values from the literature. Block B is calibrated using steady state relationships. Block C is estimated using a simulated method of moments procedure.

Block B. *Leverage* A good data counterpart to aggregate leverage in the model is hard to come by. From 2009 to 2019, the US commercial banking sector had an average leverage of 9.4.[2] This measure excludes non-bank financial institutions such as hedge funds and broker dealers that are typically more leveraged. In 2021, estimates for the total assets of the non-bank financial sector were 1.86 times larger than the total assets of commercial banks. Moreover, from 2009 to 2019, leverage of the non-financial corporate business sector was 1.9, implying a significantly lower economy-wide leverage ratio. We follow Gertler and Karadi (2011), aggregating across these highly heterogeneous sectors and, assuming that leverage in the non-bank financial sector is twice that of the commercial banking sector, end up with a conservative estimate of aggregate leverage of 3.6. Given the uncertainty in these calculations, we opt to calibrate the model to a leverage ratio of 4 (see below for further details on how we construct our measure of leverage).

Figure B1: Credit spreads and reserves in the US



Note: (a) AAA and BAA are Moody's Seasoned AAA and BAA Corporate Bond Yields, respectively; FFR is the Effective Federal Funds Rate; 10Y Tr is the market yield on Treasury Securities at 10-Year Constant Maturity. (b) Total reserves of depository institutions over total deposits of commercial banks. Source: Federal Reserve Bank of St Louis.

[2]Consistent with the model, leverage is $A/(A-L)$, where A is total assets and L is total liabilities.

Block B. ***Credit spreads*** Calibrating the steady state credit spread is equally tricky. In Figure B1(a) we plot three alternative spread measures used in the literature. The first is the spread between the BAA corporate bond yield and the federal funds rate (light blue-dot). The two component interest rates that compromise the spread are a reasonable match for the expected return on capital and the short-term policy rate in the model, respectively. We thus use the cyclical properties of these series in the estimation stage below. However, for matching the steady state credit spread, this measure is not ideal because it contains a maturity mismatch. The corporate bonds yields are based on long-term bonds with a maturity of 20 years and above whereas the federal funds rate is a short-term rate. Thus, this series is likely to contain both a liquidity and term premium in addition to a pure risk premium. To get a sense of these various premia, we plot the spread between the BAA corporate bond yield and the 10 year Treasury yield (dark blue-dash) and between the BAA and AAA corporate bond yields (green dot-dash), respectively. For the credit spread in the model, we match its steady state to 1% annualized which corresponds to the mean of the "BAA-AAA" series over the sample period. This series is generally perceived to be a good empirical measure of the safety or quality premium that we capture with the financial friction in our model (see Krishnamurthy and Vissing-Jorgensen, 2012).

Block B. ***Reserve ratio*** We set the reserve-to-deposit rate $\alpha = 0.2$. This value is broadly in line with data for both the euro area—as displayed in Figure 1—and the United States. Figure B1(b) shows the evolution of the US reserve ratio. In the aftermath of the 2007/08 financial crisis, total reserve holdings strongly increased, reflecting banks' desire to hedge against heightened liquidity risk and the Federal Reserve's willingness to supply extensive additional reserves to the banking system via a range of liquidity and QE programs. Accordingly, the reserve-to-deposit ratio rose from a pre-crisis level of around 1% to a peak of 27.9% in August 2014. The banking system's demand for liquidity spiked again during the Covid-19 crisis when the Federal Reserve once more sharply increased the provision of reserves to meet this additional demand. Overall, we find a value of 18.9% for the average reserve ratio over the post-financial crisis period in the US. As the strength of the costly interest margin channel of negative interest rates will depend sensitively on the quantity of reserves in the banking system, in Section 3.4 in the main text we conduct a sensitivity analysis where we vary this quantity and show the implications on the effectiveness of a negative interest rate policy.

Block C. We estimate the structural parameters in Block C following the method of simulated moments in Basu and Bundick (2017). In particular, the parameter values are chosen to minimize the distance between the model implied moments and their data counterparts. Formally, the vector of estimated parameters, Θ, is the solution to

$$\min_{\Theta} \left(\mathcal{H}^D - \mathcal{H}\left(\Theta\right)\right)' \mathcal{W}^{-1} \left(\mathcal{H}^D - \mathcal{H}\left(\Theta\right)\right), \tag{B36}$$

where $\mathcal{H}^D$ is a vector of data moments, $\mathcal{H}(\Theta)$ denotes its model counterpart, and $\mathcal{W}$ is a diagonal weighting matrix containing the standard errors of the estimated data moments.

The estimation targets ten moments from US time-series data and five yield curve moments. The first ten moments are the standard deviations and autocorrelations of output, consumption, inflation, the federal funds rate, and the credit spread, respectively. The remaining five moments are the movements in the 6-month, 1-, 2-, 5-, and 10-year risk-free rates, respectively, relative to the movement in the 3-month risk-free rate in response to a monetary shock. Empirical estimates are taken from Altavilla et al. (2019). The risk-free yield curve can be extracted from the model using the following set of equations:

$$\begin{aligned} P_{2,t} &= \mathbb{E}_t \Lambda_{t,t+1} P_{1,t+1}, \\ &\vdots \\ P_{40,t} &= \mathbb{E}_t \Lambda_{t,t+1} P_{39,t+1}, \end{aligned}$$

where $P_{1,t} = 1/R_t$ is the price of a 1-period risk-free bond that pays 1 unit in period $t+1$. The annualized yield on the 10-year risk-free bond is therefore given by $R_{40,t} = P_{40,t}^{-1/10}$.

With 15 moments, we estimate four parameters $\theta = \{\eta, \rho, \sigma_\zeta, \sigma_\epsilon\}$, the inverse investment elasticity, the policy rule inertia coefficient, and the standard deviations of risk premium and cost-push innovations. The estimation is thus over-identified. We choose to estimate the investment elasticity parameter because its value is not well-informed by the literature and its value has implications for the strength of the financial accelerator and the dynamics of credit spreads and net worth. The estimation delivers an inverse investment elasticity of $\eta = 1.617$. We also choose to estimate the policy rule inertia coefficient because it is crucial for the strength of the signalling channel of negative interest rates. The estimation delivers a value of $\rho = 0.856$, which suggests a significant amount of policy smoothing.

Table B1 compares the parameterized model implied moments with those from the data. The table also includes the 95% confidence interval around the data estimates. Despite only estimating a small number of parameters, the model does a good job of matching the data. The model implied moments are within the confidence interval for the yield curve moments. In terms of the business cycle moments, the model does well in terms of matching most of the standard deviations but generates too much persistence relative to the data (the exception is the credit spread, in which the data is more persistent).

Table B1: Simulated method of moments results

	Data	Model		Data	Model		Data	Model
std(y)	1.014 (0.76-1.27)	0.877	ac(y)	0.874 (0.82-0.93)	0.973	mp(r_{6m})	0.843 (0.80-0.89)	0.839
std(c)	0.714 (0.54-0.89)	0.641	ac(c)	0.831 (0.77-0.89)	0.990	mp(r_{1y})	0.677 (0.55-0.81)	0.587
std(π)	0.175 (0.14-0.21)	0.196	ac(π)	0.330 (0.14-0.52)	0.760	mp(r_{2y})	0.503 (0.29-0.72)	0.301
std(r)	0.265 (0.20-0.33)	0.144	ac(r)	0.935 (0.89-0.98)	0.961	mp(r_{5y})	0.324 (0.11-0.54)	0.135
std(cs)	0.279 (0.20-0.36)	0.345	ac(cs)	0.895 (0.83-0.95)	0.745	mp(r_{10y})	0.092 (-0.08-0.26)	0.101
Untargeted moments								
std(i)	4.470 (2.92-6.02)	4.272	ac(i)	0.914 (0.84-0.99)	0.972	cr(y, c)	0.807 (0.72-0.89)	0.599
cr(y, i)	0.906 (0.86-0.95)	0.890	cr(y, π)	0.362 (0.14-0.58)	-0.539	cr(y, r)	0.689 (0.56-0.82)	-0.644
cr(y, cs)	-0.690 (-0.84–0.54)	-0.539						

Note: Construction of moments given in Appendix B.3. y, c, π, r, and cs refer to GDP, consumption, inflation, the federal funds rate, and the credit spread, respectively. std($\cdot$) and ac($\cdot$) refer to the standard deviation and first-order autocorrelation. $r_{6m}, r_{1y}, r_{2y}, r_{5y}$, and r_{10y} refers to the OIS 6 month, 1, 2, 5, and 10 year rate, respectively. mp($\cdot$) refers to the relative response of the relevant OIS rate to the 3 month OIS rate in response to a monetary policy shock. Estimates are taken from Altavilla et al. (2019).

Data sources. We use US quarterly data covering the period 1985:Q1 to 2019:Q1. All macroeconomic and financial time series used are extracted from the Federal Reserve Economic Data (FRED) database at the St Louis FED. Table B2 summarizes this.

Data treatment We transform all nominal aggregate quantities into real per-capita terms. Inflation is defined as the quarter-on-quarter log growth rate of the GDP deflator. Nominal interest rates and spreads are divided by four to generate quarterly rates. For the estimation, all variables are stationarized using a standard HP-filter ($\lambda = 1600$). Data moments are matched with model moments for all relevant observables, where a lower case denotes the log deviation of the corresponding variable from steady state. Table B3 documents the data transformations in detail.

Table B2: Data sources

Mnemonic	Description
CNP16OV	Population level
GDP	Gross domestic product
GDPDEF	Gross domestic product: implicit price deflator
GPDI	Gross private domestic investment
PCDG	Personal consumption expenditures: durable goods
PCND	Personal consumption expenditures: nondurable goods
PCESV	Personal consumption expenditures: services
FEDFUNDS	Effective federal funds rate
DGS10	10-Year Treasury constant maturity rate
AAA	Moody's seasoned Aaa corporate bond yield
BAA	Moody's seasoned Baa corporate bond yield
TOTRESNS	Total reserves of depository institutions
DPSACBM027NBOG	Deposits, all commercial banks
TABSNNCB	Total assets, nonfinancial corporate business
TLBSNNCB	Total liabilities, nonfinancial corporate business
TLAACBW027SBOG	Total assets, all commercial banks
TLBACBW027SBOG	Total liabilities, all commercial banks

Table B3: Data treatment

Observable	Description	Construction
Steady state calibration & Figure B1		
	Spread measure I	BAA - FEDFUNDS
	Spread measure II	BAA - DGS10
	Spread measure III	BAA - AAA
	Reserve ratio	TOTRESNS/DPSACBM027NBOG
	Leverage	*see computation below**
Dynamic moment matching		
y	Output	HP-filter[GDP/(GDPDEF x NCP160V)]
c	Consumption	HP-filter[(PCND + PCESV)/(GDPDEF x NCP160V)]
π	Inflation	HP-filter[ln(GDPDEF/GDPDEF$_{-1}$)]
r	Reserve rate	HP-filter[FEDFUNDS/4]
cs	Credit spread	HP-filter[(BAA - FEDFUNDS)/4]
i	Investment	HP-filter[(PCDG + GPDI)/(GDPDEF x NCP160V)]

* Construction of the leverage series:

$$\text{Aggregate Leverage}_t = \frac{A_t^{\text{cb}}(1+s) + A_t^{\text{nfc}}}{A_t^{\text{cb}}(1+s) + A_t^{\text{nfc}} - L_t^{\text{cb}} - L_t^{\text{ncbfi}} - L_t^{\text{nfc}}}, \tag{B37}$$

where A_t and L_t denote assets and liabilities and where the superscripts "cb", "nfc", and "ncbfi" refer to commercial banks, non-financial corporations, and non-commercial bank financial institutions, respectively. L_t^{ncbfi} is given by

$$L_t^{\text{ncbfi}} = sA_t^{\text{cb}}\left(1 - \frac{1}{f\left(\frac{A_t^{\text{cb}}}{A_t^{\text{cb}} - L_t^{\text{cb}}}\right)}\right) \tag{B38}$$

where $s = 1.86$ and we assume $f = 2$.[3]

[3] The scaling factor s is derived from the May 2021 Federal Reserve Financial Stability Report, Chapter 3, Table 3. We calculate $s = A/B$ where A is the total assets of mutual funds, insurance companies, hedge funds, and broker-dealers and B is the total assets of banks and credit unions.

B.4 Parameterization: policy smoothing [Section 3.2]

In the estimation, we find a policy inertia coefficient of $\rho = 0.856$, suggesting that policy smoothing is an important feature of the data. As the strength of the signalling channel of negative interest rates will depend sensitively on the degree of policy inertia, we support the results of this estimation with further evidence, and—as for the reserve-to-deposit ratio—show sensitivity results in Section 3.4 in the main text.

Literature Figure B2(a) documents estimates of policy smoothing from the literature for the US, euro area, and four additional countries. Two key messages emerge. First, there is robust evidence for a large inertial component of monetary policy, irrespective of the estimation technique or country considered. Second, the estimates range from 0.80 (Primiceri et al., 2006, US) to 0.96 (Smets and Wouters, 2003, euro area). Thus, our baseline value of $\rho = 0.856$ is, if anything, on the more conservative side of possible parameterizations in terms of quantifying the strength of the signalling channel.[4]

Negative rates in Sweden There might be a concern that that these estimates are limited to periods in which policy rates were in positive territory. Figure B2(b) provides suggestive evidence from Sweden that policy inertia extends to negative rate episodes as well. Between February 2015 and February 2016, the Swedish Riksbank lowered the repo rate, its key policy rate, in four steps from 0% to −0.5%. Repo rate forecasts published by the Riksbank around the respective monetary policy decisions show that every negative rate decision came with a substantial downward revision of the forecasted path of the future policy rate, both extending the expected ZLB duration and lowering the expected future policy rate. This is consistent with inertial policy-setting as documented above.

[4] Rudebusch (2002, 2006) argues that observed policy inertia may, in fact, reflect persistent shocks rather than interest rate smoothing. However, recent work by Coibion and Gorodnichenko (2012) finds strong evidence in favour of the interest rate smoothing explanation.

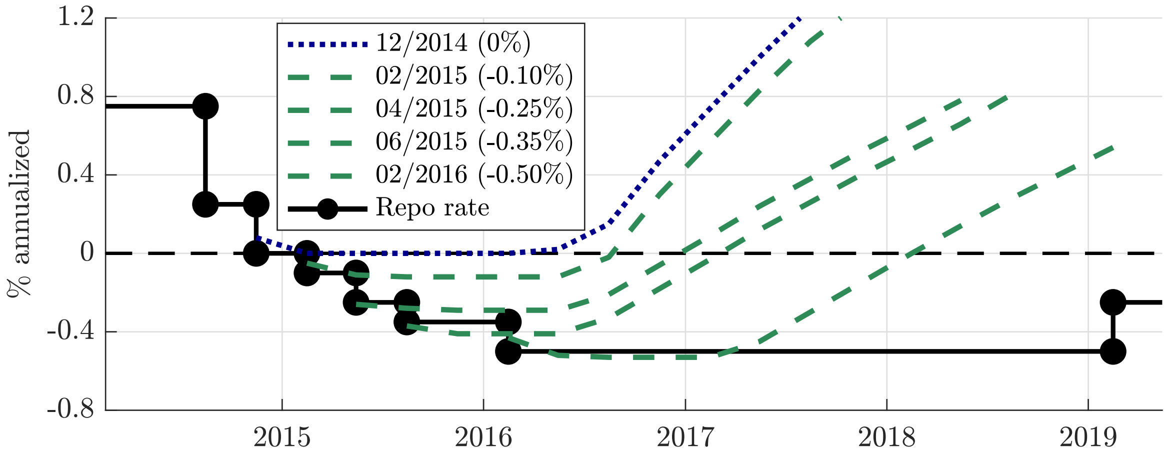
Figure B2: Monetary policy inertia in the literature and in practice

(a) Estimates of policy rule inertia

United States		**Euro area**	
Primiceri et al. (2006)	0.80	Smets and Wouters (2003)	0.96
Smets and Wouters (2007)	0.81	Christiano et al. (2010)	0.84
Coibion and Gorodnichenko (2012)	0.83	Darracq Pariès et al. (2011)	0.84
Brayton et al. (2014)	0.85	Coenen et al. (2018)	0.93
Christiano et al. (2014)	0.85	**Japan**	
United Kingdom		Sugo and Ueda (2007)	0.84
Burgess et al. (2013)	0.83	**Sweden**	
Switzerland		Adolfson et al. (2008)	0.88
Rudolf and Zurlinden (2014)	0.90	Christiano et al. (2011)	0.82

Note: Estimates of ρ for a selection of papers and central bank policy models. Brayton et al. (2014) is the Federal Reserve's FRB/US model, Burgess et al. (2013) is the Bank of England's COMPASS model, and Coenen et al. (2018) is the ECB's New Area Wide Model II.

(b) Riksbank repo rate forecasts during negative interest rates

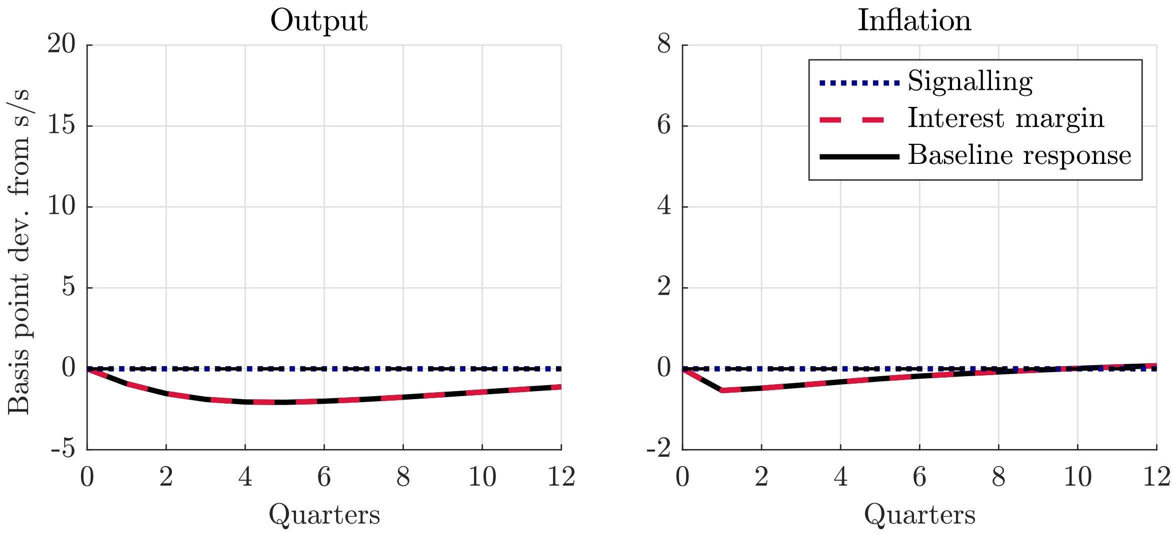


Note: The blue-dot and green-dash lines show the Riksbank's own repo rate forecasts around monetary policy meetings in which they lowered the repo rate, based on quarterly averages. The actual repo rate (black-solid line) is based on daily data. Source: Riksbank monetary policy reports.

B.5 Results: signalling channel without inertia [Section 3.3]

Figure B3 is analogous to Figure 6 in the main text and shows the signalling channel vs. net interest margin channel decomposition when $\rho = 0$ in the quantitative model. The figure shows that in the absence of r_{t-1} in the monetary policy rule, the signalling channel is completely shut down. This is true despite the existence of a range of further endogenous state variables in the model.

Figure B3: Contribution of signalling and interest margin channels (**no inertia**)



Note: Replication of Figure 6 without policy inertia ($\rho = 0$). Impulse responses to a -25bp iid monetary policy shock at the ZLB. Inflation is annualized. We linearly decompose the baseline response into "Signalling"—$\alpha = 0$ and $\rho = 0$, i.e. no costly interest margin channel—and "Interest margin"—difference between the baseline and "Signalling".

B.6 Results: derivation of bank profit decomposition [Section 3.3]

Baseline This section derives Equation (30) in the main text. From Equations (27) and (28), the evolution of net worth (conditional on not exiting) is given by

$$N_t = \left(R_{k,t}\Phi_{t-1} - \frac{R_{d,t-1} - \alpha(x_t) R_{t-1}}{(1-\alpha(x_t))\Pi_t} (\Phi_{t-1} - 1) \right) N_{t-1}. \tag{B39}$$

Defining profits as $\mathtt{prof}_t \equiv \Pi_t N_t / N_{t-1}$ and rearranging terms gives

$$\mathtt{prof}_t = (\Pi_t R_{k,t} - R_{d,t-1})\Phi_{t-1} + R_{d,t-1} - \frac{\alpha(x_t)}{1-\alpha(x_t)} (R_{d,t-1} - R_{t-1})(\Phi_{t-1} - 1). \tag{B40}$$

Adding and subtracting $\mathbb{E}_{t-1}\Pi_t R_{k,t}\Phi_{t-1}$ gives

$$\begin{aligned} \mathtt{prof}_t = & \left(\Pi_t \frac{\mathtt{mpk}_t + Q_t - \delta}{Q_{t-1}} - \mathbb{E}_{t-1}\Pi_t \frac{\mathtt{mpk}_t + Q_t - \delta}{Q_{t-1}} \right) \Phi_{t-1} \\ & + \mathtt{cs}_{t-1}\Phi_{t-1} + R_{d,t-1} - \frac{\alpha(x_t)}{1-\alpha(x_t)} (R_{d,t-1} - R_{t-1})(\Phi_{t-1} - 1), \end{aligned} \tag{B41}$$

where $R_{k,t} = \frac{\mathtt{mpk}_t + Q_t - \delta}{Q_{t-1}}$ and $\mathtt{cs}_t \equiv \mathbb{E}_t \Pi_{t+1} R_{k,t+1} - R_{d,t}$.
Log-linearizing and collecting terms we arrive at Equation (30) in the main text.

Model with firm equity and loan finance Suppose instead that firms borrow from banks using a combination of equity and loans in proportion $\mathtt{s}$ and $1-\mathtt{s}$, respectively. In particular, suppose that the return to a banker on a unit of capital is given by

$$R_{s,t} = \mathtt{s} R_{k,t} + (1-\mathtt{s}) R_{l,t-1}, \tag{B42}$$

where $R_{l,t} \equiv \mathbb{E}_t R_{k,t+1}$. In this case, the credit spread is $\mathtt{cs}_t \equiv \mathbb{E}_t \Pi_{t+1} R_{s,t+1} - R_{d,t}$ and the first-term on the right-hand side of Equation (B40) becomes $(\Pi_t R_{s,t} - R_{d,t-1})\Phi_{t-1}$.

Adding and subtracting $\mathbb{E}_{t-1}\Pi_t R_{s,t}$ gives

$$(\Pi_t R_{s,t} - \mathbb{E}_{t-1}\Pi_t R_{s,t})\Phi_{t-1} + \mathtt{cs}_{t-1}\Phi_{t-1}. \tag{B43}$$

Log-linearizing around the deterministic steady state gives

$$R_s\Phi(\hat{\pi}_t - \mathbb{E}_{t-1}\hat{\pi}_t) + R_s\Phi(\hat{r}_{s,t} - \mathbb{E}_{t-1}\hat{r}_{s,t}) + \mathtt{cs}\Phi\left(\hat{\mathtt{cs}}_{t-1} + \hat{\phi}_{t-1}\right). \tag{B44}$$

while log-linearizing $R_{s,t}$ gives

$$R_s\hat{r}_{s,t} = \mathtt{s}\left(\mathtt{mpk}\cdot\hat{\mathtt{mpk}}_t + \hat{q}_t\right) + (1-\mathtt{s})\mathbb{E}_{t-1}\left(\mathtt{mpk}\cdot\hat{\mathtt{mpk}}_t + \hat{q}_t\right) - R_k\hat{q}_{t-1}. \tag{B45}$$

Therefore

$$R_s\left(\hat{r}_{s,t}-\mathbb{E}_{t-1}r_{s,t}\right)=\mathtt{s}\cdot\mathtt{mpk}\left(\hat{\mathtt{mpk}}_t-\mathbb{E}_{t-1}\hat{\mathtt{mpk}}_t\right)+\mathtt{s}\left(\hat{q}_t-\mathbb{E}_{t-1}\hat{q}_t\right). \tag{B46}$$

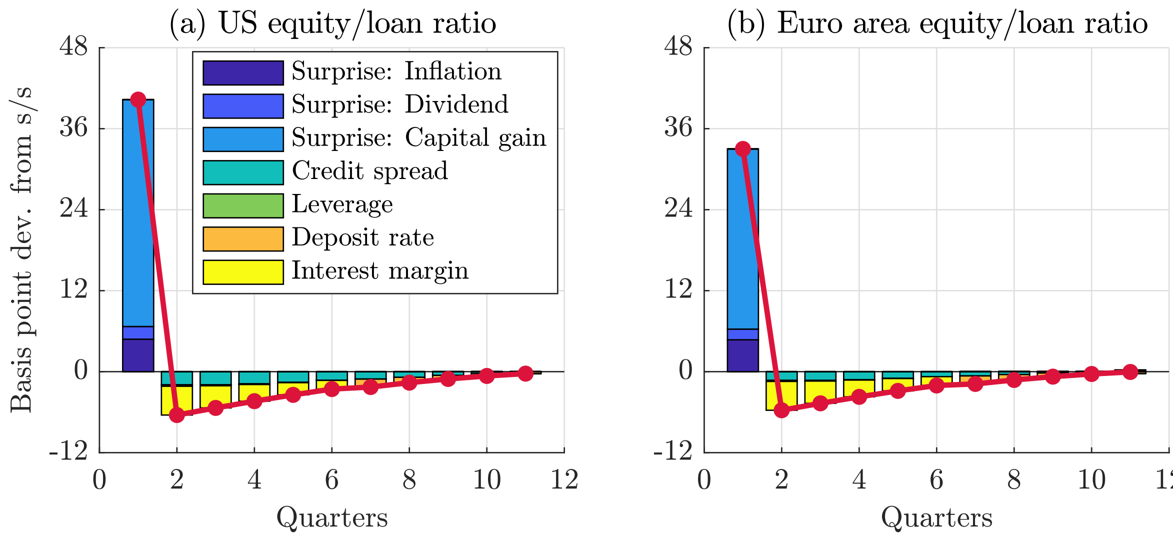
Finally, the augmented version of Equation (30) that accounts for bank assets being composed of a mix of equity and loans is given by

$$\begin{aligned}\hat{\mathtt{prof}}_t=&\underbrace{\frac{R_k\Phi}{\mathtt{prof}}\left(\hat{\pi}_t-\mathbb{E}_{t-1}\hat{\pi}_t\right)}_{\text{Surprise: Inflation}}+\underbrace{\mathtt{s}\frac{\mathtt{mpk}\Phi}{\mathtt{prof}}\left(\hat{\mathtt{mpk}}_t-\mathbb{E}_{t-1}\hat{\mathtt{mpk}}_t\right)}_{\text{Surprise: Dividend}}+\underbrace{\mathtt{s}\frac{\Phi}{\mathtt{prof}}\left(\hat{q}_t-\mathbb{E}_{t-1}\hat{q}_t\right)}_{\text{Surprise: Capital gain}}\\&+\underbrace{\frac{\mathtt{cs}\Phi}{\mathtt{prof}}\hat{\mathtt{cs}}_{t-1}}_{\text{Credit spread}}+\underbrace{\frac{\mathtt{cs}\Phi}{\mathtt{prof}}\hat{\phi}_{t-1}}_{\text{Leverage}}+\underbrace{\frac{R_d}{\mathtt{prof}}\hat{r}_{d,t-1}}_{\text{Deposit rate}}-\underbrace{\frac{\alpha}{1-\alpha}\frac{R_d\left(\Phi-1\right)}{\mathtt{prof}}\left(\hat{r}_{d,t-1}-\hat{r}_{t-1}\right)}_{\text{Interest margin channel}}.\end{aligned} \tag{B47}$$

When $\mathtt{s}=1$, the formulation is the same as Equation (30) in the main text.

Based on De Fiore and Uhlig (2011) though, the debt to equity ratio of the non-financial sector is 0.43 in the US and 0.64 in the euro area. This translates to $\mathtt{s}_{US}=1/1.43\approx 0.70$ and $\mathtt{s}_{EA}=1/1.64\approx 0.61$, respectively. Figure B4 supplements our analysis on the robustness of our model to changes in the firm equity-to-loan ratio showing the results of the bank profit decomposition in Figure 6 in the main text for $\mathtt{s}_{US}$ and $\mathtt{s}_{EA}$.

Figure B4: Decomposition of bank profits
— Sensitivity with respect to **equity/loan ratio** —



NOTE: Replication of Figure 7 for alternative firm equity/loan ratios $\mathtt{s}$. (a) $\mathtt{s}_{US}=0.70$, (b) $\mathtt{s}_{EA}=0.61$. $\alpha=0.2$, $\rho=0.85$. The red-dot line plots the impulse response of bank profits to a -25bp iid monetary policy shock at the ZLB. Stacked bars decompose the impulse response for every period.

B.7 Results: further robustness [Section 3.4]

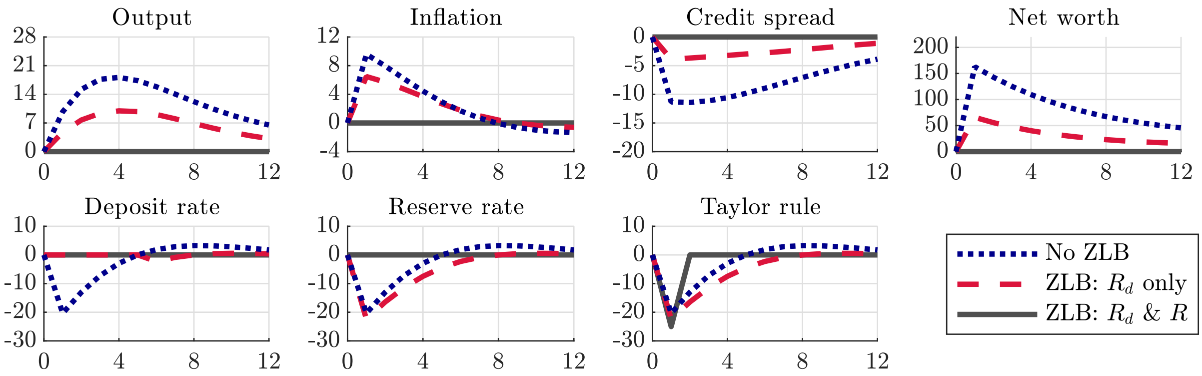
This section provides additional details and reports the results for the further robustness exercises summarized towards the end of Section 3.4 in the main text.

Sensitivity w.r.t. Frisch elasticity As Chetty et al. (2011) report, macro estimates of the Frisch elasticity from real business cycle models range from 2.61 to 4. Our baseline value of 3.75 is within this range. Smets and Wouters (2007) estimate the value to be 1.92 whereas micro estimates are around 0.82. Another popular choice in the literature for calibrated models is to set the elasticity to 1, between the micro and macro estimates as in Hazell et al. (2022). Figure B5 replicates Figure 5 in the main text for a range of plausible empirical values of the inverse elasticity of labor supply/ Frisch elasticity φ. The figure shows that our results regarding the effectiveness of negative interest rates are robust to changes of the exact value of φ. A higher inverse Frisch elasticity (i.e. a lower labor supply elasticity of the household in the model) reduces the expansion in output in response to a monetary policy easing relative to the baseline (depicted in rows 5 and 6). However, since this is true for both monetary policy surprises in normal times and at the ZLB, the relative efficiency of a monetary policy easing into negative territory remains broadly unchanged (as can be seen comparing the red and blue lines across specifications).

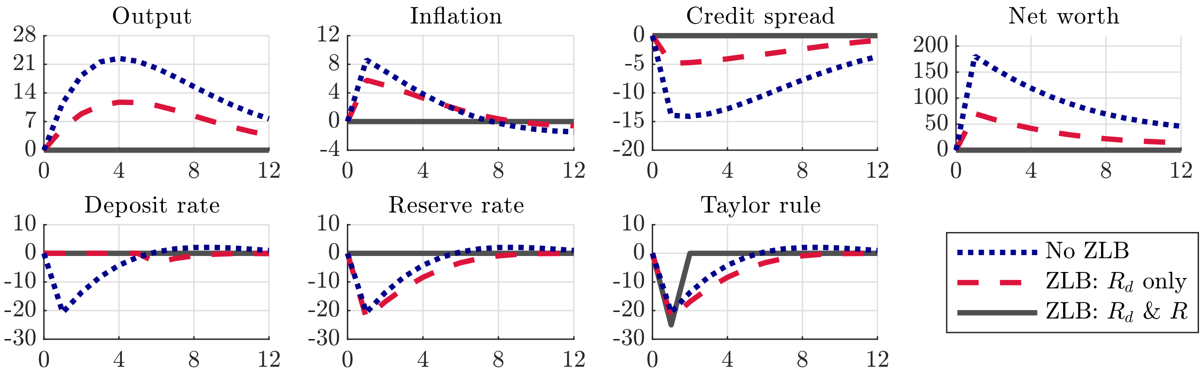
Sensitivity w.r.t. Phillips curve slope As Harding et al. (2022) report, estimates of the new-Keynesian Phillips curve slope in the literature range from 0.009 to 0.014. Hazell et al. (2022) estimate the unemployment-inflation slope to be 0.0062. Based on a Frisch elasticity between 1 and 3.62 (as above), this gives a Phillips curve slope in the range $0.006 - 0.023$. With a Calvo parameter of 0.9, our baseline Phillips curve slope is 0.012, well within both ranges. Figure B6 replicates Figure 5 in the main text for different combinations of plausible empirical values of the inverse elasticity of labor supply/ Frisch elasticity φ and the Calvo parameter ι keeping the unemployment-inflation slope constant at 0.0062 as suggested by Hazell et al. (2022). The figure shows that our results regarding the effectiveness of negative interest rates are robust to changes in the slope of the new-Keynesian Phillips Curve. A higher inverse Frisch elasticity paired with tighter price rigidity (i.e. a larger Calvo parameter) reduces both the expansion in output and inflation in response to a monetary policy easing relative to the baseline case. However—as in the case where we just vary the Frisch elasticity φ—since this is true for monetary policy surprises in normal times and at the ZLB, the relative efficiency of a monetary policy easing into negative territory remains broadly unchanged.

Figure B5: Monetary policy shock with inertia in the policy rule
— Sensitivity with respect to **inverse Frisch elasticity** —

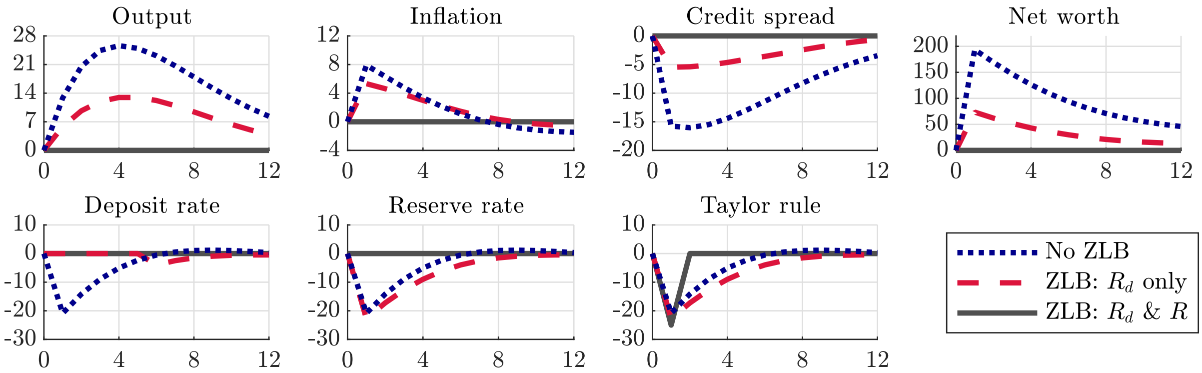
(a) High inverse Frisch elasticity



(b) Medium inverse Frisch elasticity



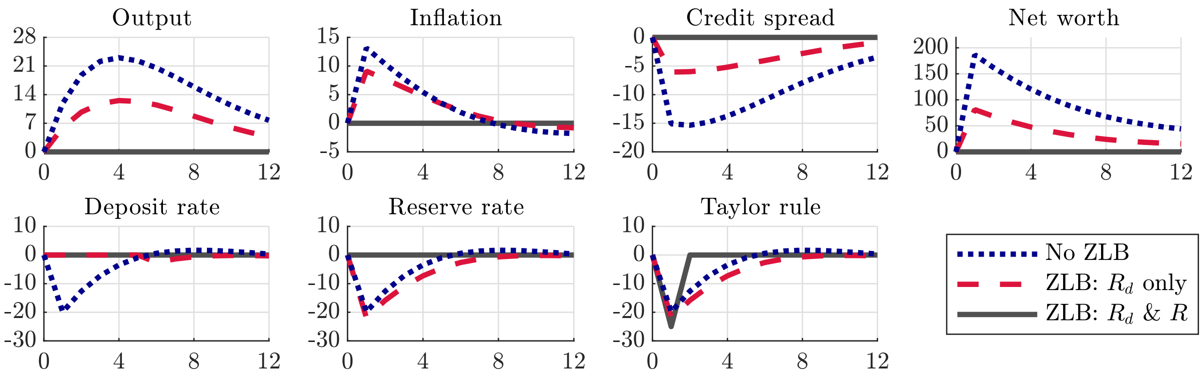
(c) Baseline inverse Frisch elasticity



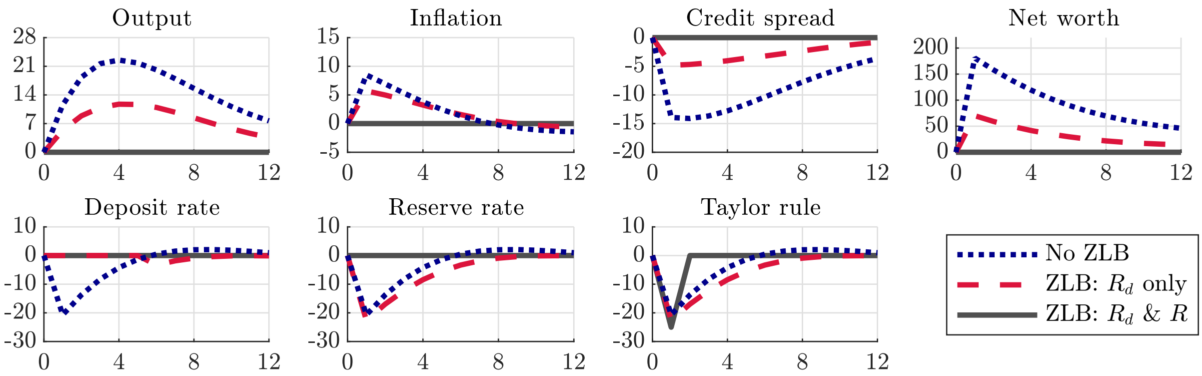
Note: $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a -25bp iid monetary policy shock at the ZLB. Rows 1 and 2 show results for an inverse Frisch elasticity of $\varphi = 1$ (Hazell et al., 2022), rows 3 and 4 for $\varphi = 0.521$ (Smets and Wouters, 2007), and rows 5 and 6 for $\varphi = 0.276$ (baseline). Interest rates are in annualized basis points. All other variables are in basis point deviation from steady state. Inflation is annualized.

Figure B6: Monetary policy shock with inertia in the policy rule
— Sensitivity with respect to **Phillips Curve slope** —

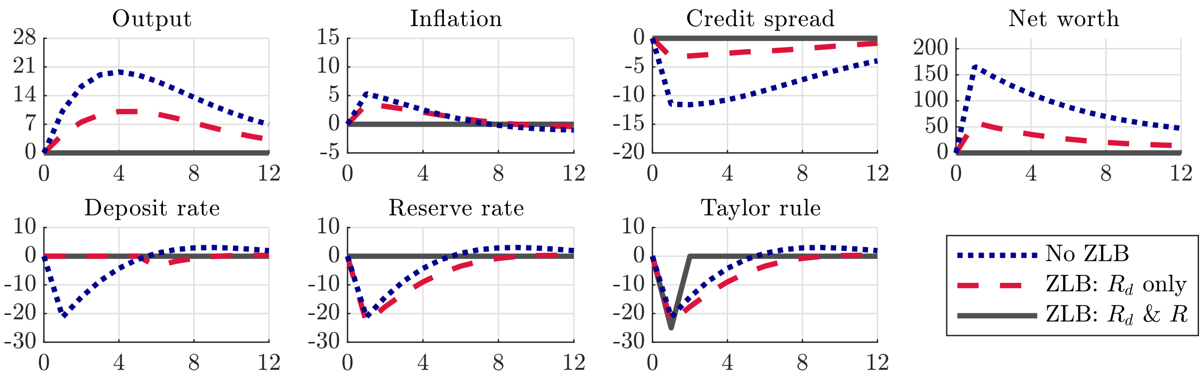
(a) Steep Phillips Curve



(b) Medium Phillips Curve



(c) Flat Phillips Curve

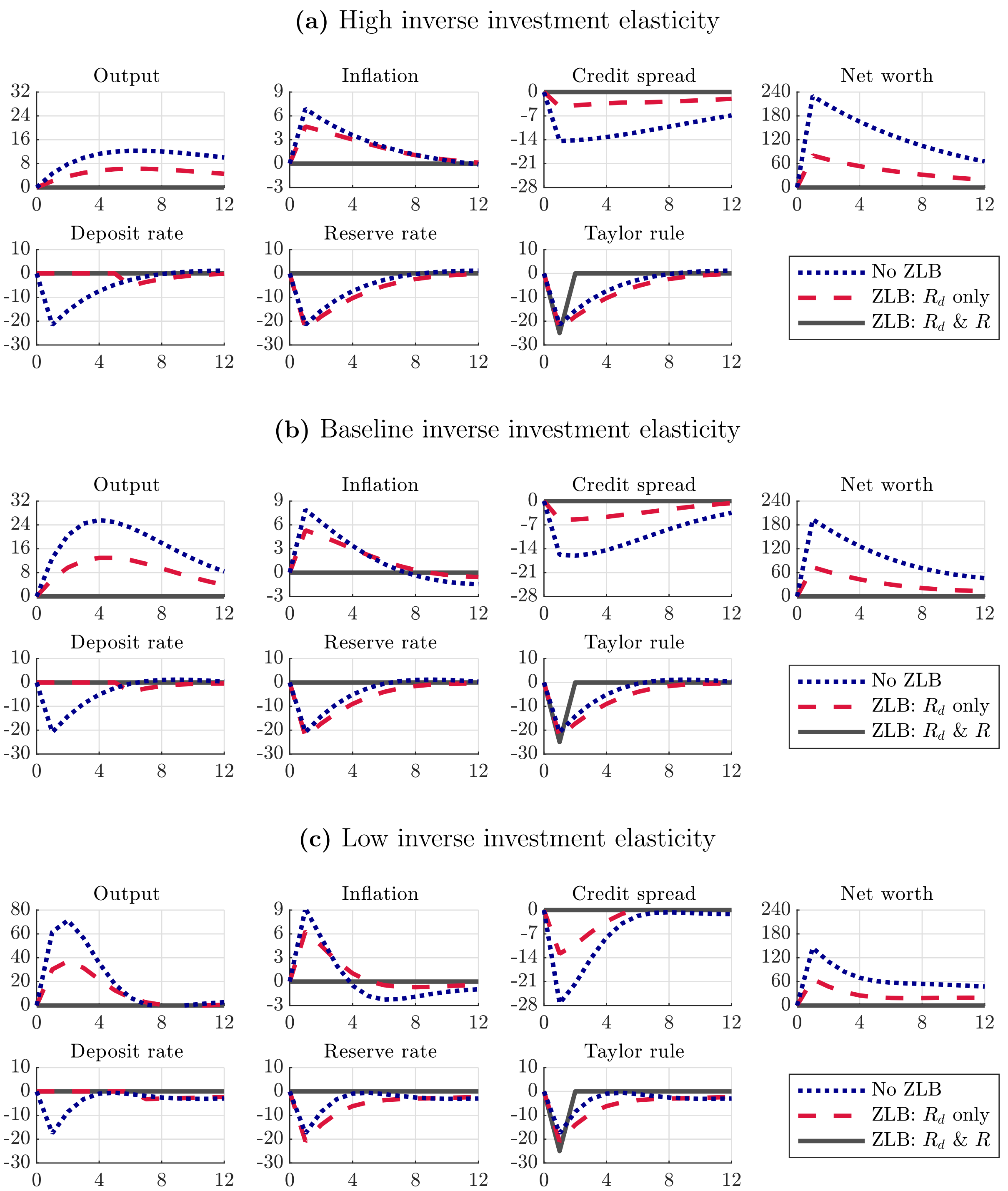


Note: $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a -25bp iid monetary policy shock at the ZLB. The values for the inverse Frisch elasticity φ from Figure B5 are paired with a Calvo parameter ι such that the unemployment-inflation trade-off in all specifications is 0.0062 as suggested by Hazell et al. (2022). Rows 1 and 2 depict results for $\{\varphi = 0.276, \iota = 0.865\}$, rows 3 and 4 for $\{\varphi = 0.521, \iota = 0.901\}$, and rows 5 and 6 for $\{\varphi = 1, \iota = 0.929\}$. Interest rates are in annualized basis points. All other variables are in basis point deviation from steady state. Inflation is annualized.

Sensitivity w.r.t. investment elasticity The inverse investment elasticity η is difficult to pin down from the literature which is why we include the parameter in the estimation matching a range of empirical moments (see Appendix B.3 for details). Our estimate of $\eta = 1.617$ is close to the 1.728 value picked by Gertler and Karadi (2011) and delivers a net worth response to an unconstrained 25bp monetary policy shock in line with the empirical response in Jarociński and Karadi (2020) (194bp versus 210bp on impact). Since the net worth response to monetary policy was not targeted in our estimation, this outcome provides external validation for our parameterization. However, as the parameter is crucial for the strength of the financial accelerator, we test the robustness of our main results for a wide parameter range in Figure B7. The figure shows that our results regarding the effectiveness of negative interest rates are robust to changes in the investment elasticity. Increasing the investment elasticity strengthens the impact response but decreases the persistence of monetary policy. Figure B8 replicates Figure 7 in the main text. In terms of banks profitability, increasing the investment elasticity decreases windfall capital gains for banks (as asset prices are less responsive) but raises windfall dividends (as investment is more responsive) to a negative rate shock.

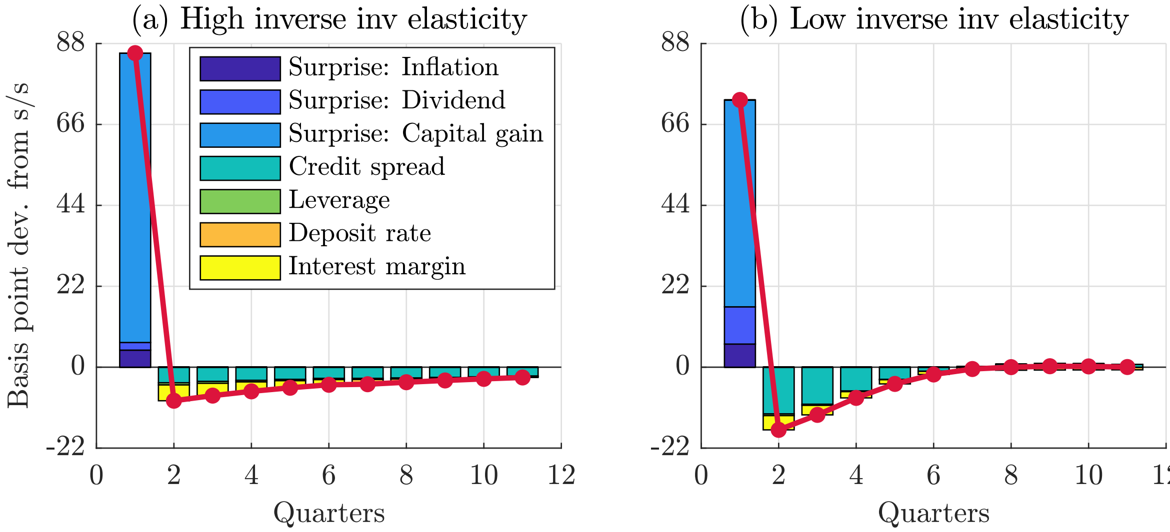
Sensitivity w.r.t. wage rigidities Finally, we augment the model with nominal wage rigidities. Figures B9-B11 replicate Figures 4-6 in the main text when nominal wage rigidities supplement nominal price rigidity in the model. The extension of the model in this dimension is straightforward. Appendix B.8 details the necessary modifications. For simplicity, we keep the baseline calibration unchanged and set the structural parameters associated with Calvo wage rigidities as follows: ϵ_w—the elasticity of substitution between different types of labor—is set to 4.167 (equal to the value picked for price rigidities), and ι_w—the probability of not being able to adjust wages next period—is set to 0.5. We also marginally increase the size of the risk premium shock that takes the model to the ZLB in order to regenerate our baseline experiment with the ZLB binding for four periods (intuitively, this adjustment is needed due to the additional persistence in the model). The figures show that a high Frisch elasticity and nominal wage rigidities are, to some degree, substitutable in our analysis and that our results regarding the effectiveness of negative interest rates are robust to the introduction of rigid wages. The introduction of an additional nominal rigidity makes monetary policy interventions more powerful. However—as in the case where we vary the Frisch elasticity and the slope of the new-Keynesian Phillips Curve—since this is true for monetary policy surprises in normal times and at the ZLB, the relative efficiency of a monetary policy easing into negative territory remains broadly unchanged. Similar results can be obtained for alternative specifications of ϵ_w and ι_w.

Figure B7: Monetary policy shock with inertia in the policy rule
— Sensitivity with respect to **inverse investment elasticity** —



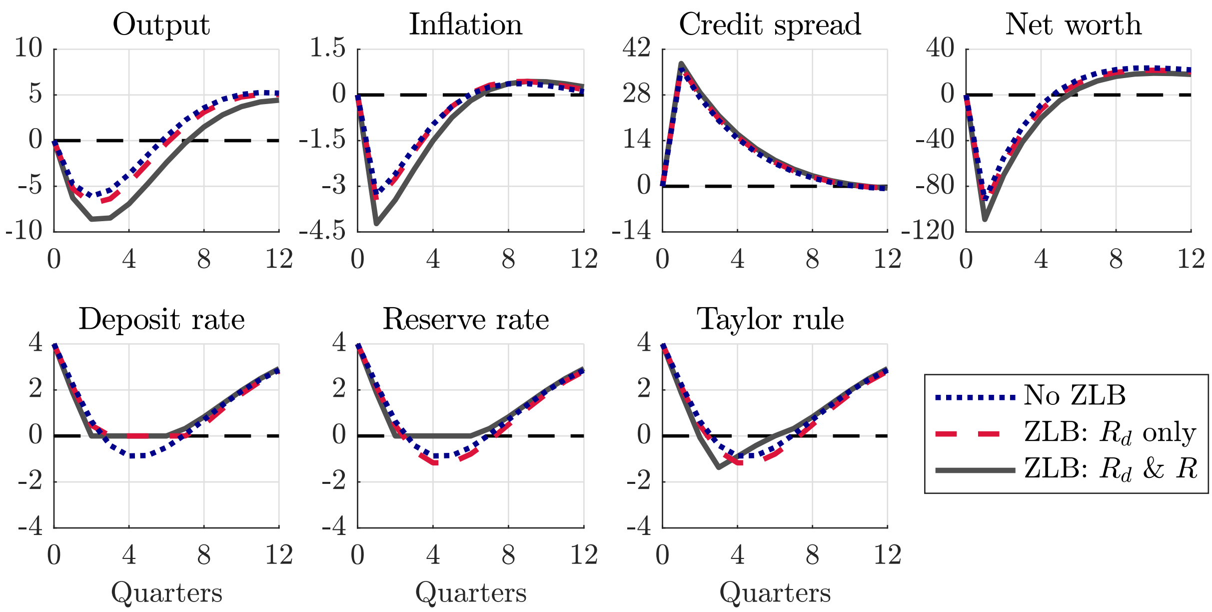
Note: $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a -25bp iid monetary policy shock at the ZLB. Rows 1 and 2 show results for an inverse investment elasticity of $\eta = 10$, rows 3 and 4 for $\eta = 1.617$ (baseline), and rows 5 and 6 for $\eta = 0.1$. Interest rates are in annualized basis points. All other variables are in basis point deviation from steady state. Inflation is annualized.

Figure B8: Decomposition of bank profits
— Sensitivity with respect to **inverse investment elasticity** —



Note: Replication of Figure 7 for alternative inverse investment elasticities, η. (a) $\eta = 10$, (b) $\eta = 0.1$. $\alpha = 0.2$, $\rho = 0.85$. The red-dot line plots the impulse response of bank profits to a -25bp iid monetary policy shock at the ZLB. Stacked bars decompose the impulse response for every period.

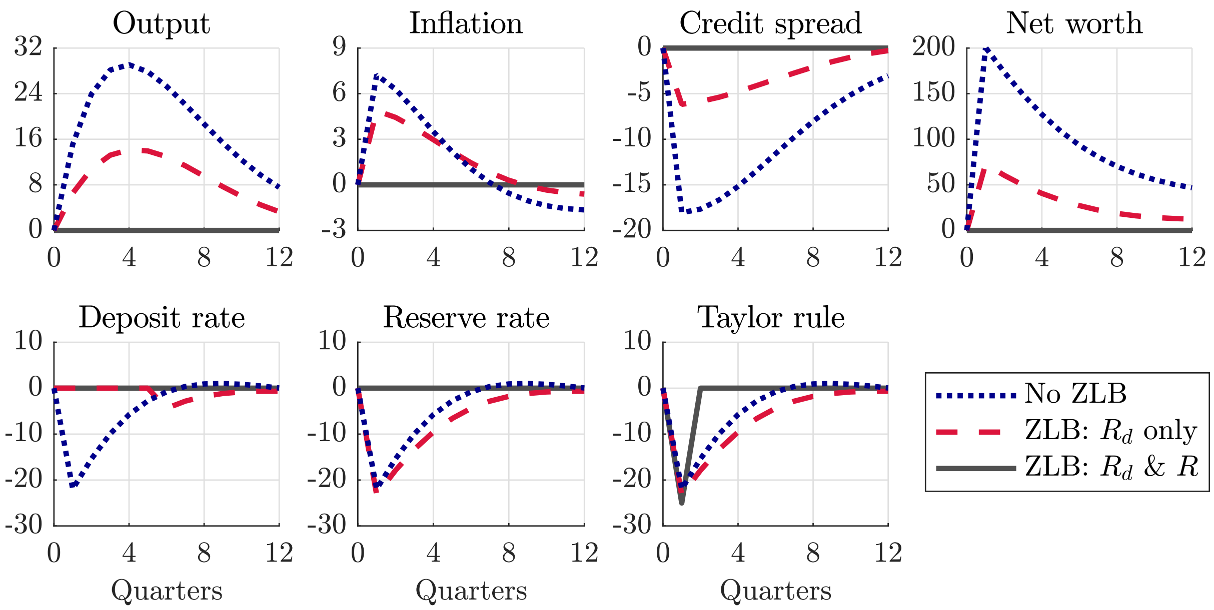
Figure B9: Wage rigidities: Risk premium shock with inertia in the policy rule



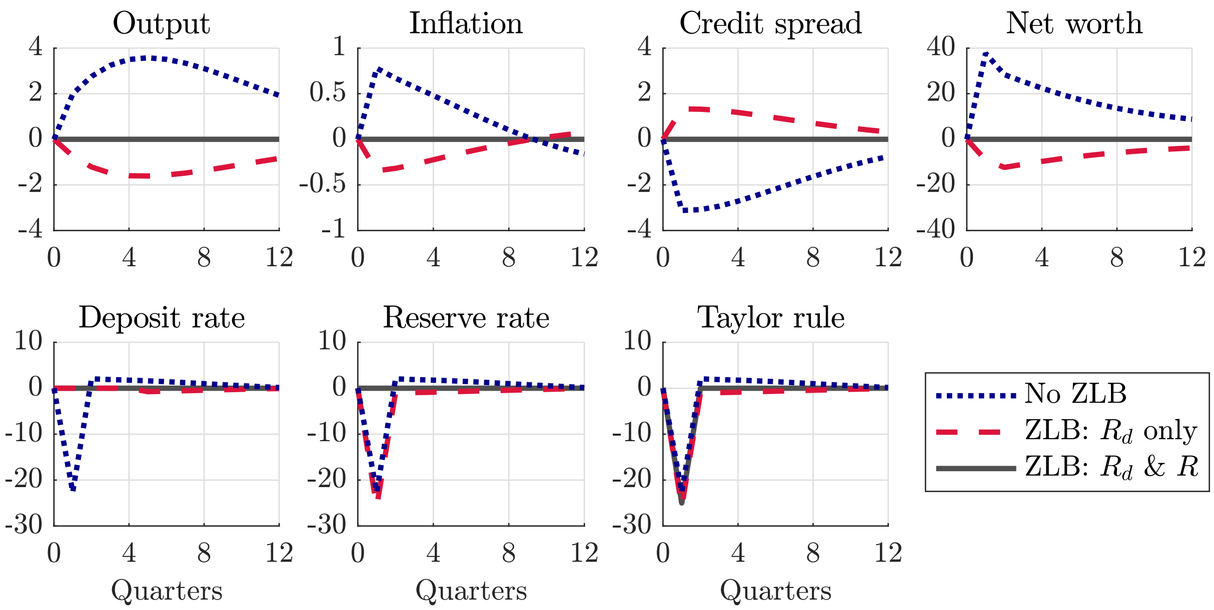
Note: Replication of Figure 4 with wage rigidities added to model ($\iota_w = 0.5$). $\alpha = 0.2$, $\rho = 0.85$. Impulse responses to a risk premium shock that brings the economy to the ZLB for 4 quarters. All interest rates displayed are in annualized percent. All other variables are in 100×log-deviation from steady state. Inflation is annualized.

Figure B10: Wage rigidities: Monetary policy shock in negative territory

(a) Policy rule with inertia

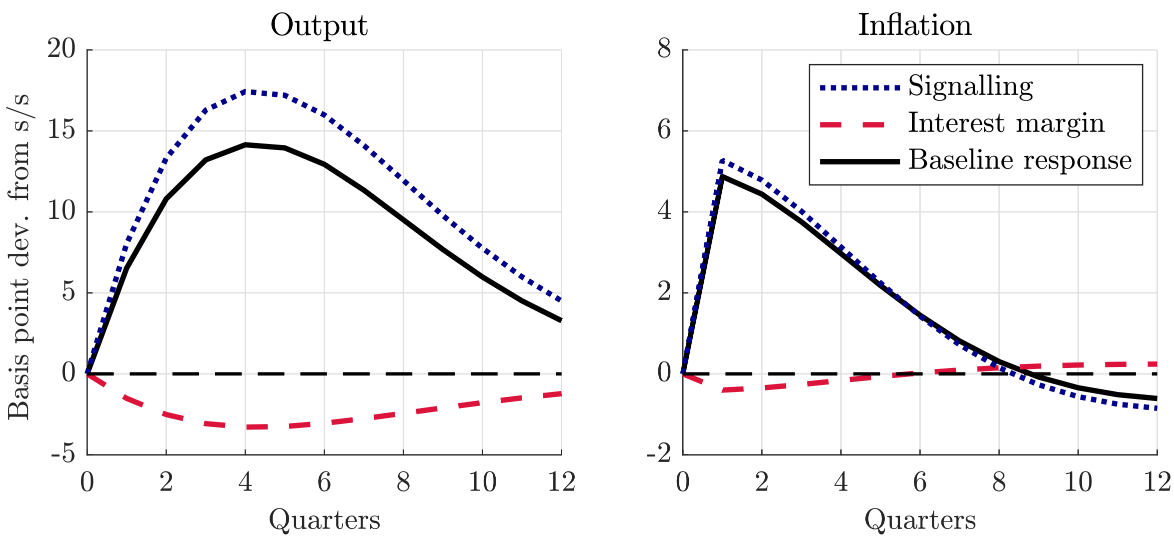


(b) Policy rule without inertia



Note: Replication of Figure 5 with wage rigidities added to model ($\iota_w = 0.5$). (a) $\alpha = 0.2$ and $\rho = 0.85$, (b)$\alpha = 0.2$ and $\rho = 0$. Impulse responses to a -25bp iid monetary policy shock at the ZLB. Interest rates are in annualized basis points. All other variables are in basis point deviation from steady state. Inflation is annualized.

Figure B11: Wage rigidities: Contribution of signalling and interest margin channels



Note: Replication of Figure 6 with wage rigidities added to model ($\iota_w = 0.5$). Impulse responses to a −25bp iid monetary policy shock at the ZLB. Inflation is annualized. We linearly decompose the baseline response into "Signalling"—$\alpha = 0$ and $\rho = 0.85$, i.e. no costly interest margin channel—and "Interest margin"—difference between the baseline and "Signalling".

B.8 Results: equilibrium with wage rigidities [Section 3.4]

We add nominal wage rigidities to the quantitative model following Erceg et al. (2000). Households supply homogeneous labor $L_{h,t}$ at price $W_{h,t}$. Monopolistic labor unions, owned by households, diversify and sell the labor good to intermediate goods firms as CES aggregate L_t at mark-up price W_t. In equilibrium, this extends the model to 27 equations in 27 endogenous variables, $\{Y_t,\ Y_{m,t},\ L_t,\ L_{h,t},\ C_t,\ \tilde{C}_t,\ \Lambda_{t,t+1},\ \mu_t,\ K_t,\ I_t,\ I_{n,t},\ N_t,\ \Phi_t,\ \Delta_t,\ \Delta_{w,t},\ W_t,\ W_{h,t},\ \Pi_t,\ \Pi_{w,t},\ X_t,\ P_{m,t},\ Q_t,\ R_{k,t},\ R_{T,t},\ R_t,\ R_{d,t},\ CS_t\}$, and 3 exogenous processes, $\{\zeta_t,\ \epsilon_t,\ \varepsilon_{m,t}\}$. In the following we only restate the equations that are new or modified relative to the overview in B.2.

Households

- Labor supply (modified)

$$\mu_t W_{h,t} = \chi L_{h,t}^{\varphi} \tag{B48}$$

Labor Unions

- Wage Phillips Curve (new)

$$\left(\frac{\epsilon_w}{\epsilon_w - 1}\right)\frac{\mathcal{D}_{w,t}}{\mathcal{F}_{w,t}} = \left[\frac{1-\iota_w\left(\Pi_t\,\Pi_{w,t}\right)^{\epsilon_w-1}}{1-\iota_w}\right]^{\frac{1}{1-\epsilon_w}} \tag{B49}$$

$$\text{where}\quad \begin{aligned}\mathcal{D}_{w,t} &\equiv \mu_t W_{h,t} L_t + \beta\iota_w \mathbb{E}_t\left(\Pi_{t+1}\,\Pi_{wt+1}\right)^{\epsilon_w}\mathcal{D}_{w,t+1},\\ \mathcal{F}_{w,t} &\equiv \mu_t W_t L_t + \beta\iota_w \mathbb{E}_t\left(\Pi_{t+1}\,\Pi_{wt+1}\right)^{\epsilon_w-1}\mathcal{F}_{w,t+1}.\end{aligned}$$

- Wage dispersion (new)

$$\Delta_{w,t} = (1-\iota_w)\left[\left(\frac{\epsilon_w}{\epsilon_w-1}\right)\frac{\mathcal{D}_{w,t}}{\mathcal{F}_{w,t}}\right]^{-\epsilon_w} + \iota_w\left(\Pi_t\,\Pi_{w,t}\right)^{\epsilon_w}\Delta_{w,t-1} \tag{B50}$$

- Wage inflation (new)

$$\Pi_{w,t} = W_t / W_{t-1} \tag{B51}$$

General equilibrium

- Aggregate labor (new)

$$L_t = L_{h,t}/\Delta_{w,t}, \tag{B52}$$

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